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Proportional damping approximation using the energy gain and simultaneous perturbation stochastic approximation

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ABSTRACT

The design of vector second-order linear systems for accurate proportional damping approximation is addressed. For this purpose an error system is defined using the difference between the generalized coordinates of the non-proportionally damped system and its proportionally damped approximation in modal space. The accuracy of the approximation is characterized using the energy gain of the error system and the design problem is formulated as selecting parameters of the non-proportionally damped system to ensure that this gain is sufficiently small. An efficient algorithm that combines linear matrix inequalities and simultaneous perturbation stochastic approximation is developed to solve the problem and examples of its application to tensegrity structures design are presented.

1. Introduction

The linearized dynamics of physical systems such as multi-body robots, structures, etc., can be described by vector second-order linear systems,

$$M\ddot{q}+C\dot{q}+Kq=f, \quad M>0, C\geq 0, K>0, \tag{1}$$

where, using a terminology that is common to mechanical systems, M, C, K are the mass, damping, stiffness matrices, q is the n-dimensional vector of generalized coordinates, and f is the vector of external loads or perturbations, respectively. Unlike inertial and stiffness characteristics, which can be easily measured in static conditions, damping, a dynamic characteristic, is more difficult to quantify. Hence, the *artificial* Rayleigh damping model, which assumes that the damping matrix is a linear combination of the mass and stiffness matrices, is frequently used. Rayleigh damping, or a generalization of it [1–3], is preferred because it leads to the ideal situation in which the system described by Eq. (1) is proportionally damped (i.e. the modal damping matrix is diagonal), but, in general, it is not a physics-based model.

When the source of damping can be identified and accurately modeled using physics principles, the Rayleigh damping assumption is not recommended. For example, in structures composed of pin-jointed bars and cables such as tensegrity [4] the major damping sources can be easily identified: the joints and the cables. For these components reliable physics-based damping models can be built [5–7]. However, even in these situations, in most cases the resulting linearized dynamics models are not proportionally damped [6,7]. In general, for many physical systems the likelihood of obtaining linearized models that are not proportionally damped will increase due to our enhanced ability to accurately model damping using

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physics principles. Note also that the viscous damping assumption is in itself a limiting approximation. In sophisticated models, damping is not expressed in viscous terms and another stage of approximation is necessary.

Even if the system described by Eq. (1) is not proportionally damped, one would still like to approximate it with a proportionally damped system, which lends itself easily to computationally efficient identification and control design tools [8]. Previous research has investigated this problem using an ‘‘a-posteriori’’ approach, in the sense that after the system had been designed and the mass, damping, and stiffness matrices had been determined, a proportionally damped approximation was constructed and the quality of the approximation evaluated using various non-proportionality measures/indices [8–11]. In this article a different idea is pursued: parameters of the physical system are used to *design* the system such that its linearized dynamics model, given by Eq. (1), is ‘‘close’’ to a proportionally damped one that can be used as an accurate approximation.

This idea was explored to a limited extent in [12] in the context of designing structures for dynamic properties. However, in [12] the system described by Eq. (1) was not specifically designed for accurate proportional damping approximation; instead, an efficient algorithm that guarantees satisfaction of certain design specifications on the natural frequencies of the system described by Eq. (1), such as prescribed separation between these frequencies, was presented and illustrated in the design of structures. Because separation between natural frequencies represents only one, *indirect* factor that influences the quality of the proportional damping approximation, it might be a misleading criterion for the accuracy of the approximation [12–14].

In this article a *direct* measure of the accuracy of the approximation is used. For this purpose, a proportionally damped linear system is built by removing the off-diagonal terms from the modal damping matrix of the non-proportionally damped system and an error system is defined using the difference between the generalized coordinates of these systems. The ‘‘distance’’ between the two systems, i.e. the accuracy of the approximation, is characterized using the energy gain of the error system, which is directly related to the approximation error: a small energy gain guarantees that the ratio between the energy of the error, defined as its L_2 norm, and the energy of external perturbations is small. Therefore, for an accurate approximation it appears natural to design the system such that the energy gain of the error system is small. A major advantage of the energy gain is that, for linear time invariant systems, reliable linear matrix inequalities solving techniques can be used to compute an upper bound on it [15]. Moreover, under general conditions (i.e. minimal realization of the system) this upper bound is actually equal to the energy gain [15]. Therefore, an iterative algorithm that uses linear matrix inequalities and adaptive simultaneous perturbation stochastic approximation [16] techniques is developed to reduce it. Applications of the algorithm to tensegrity structures design demonstrates the feasibility of the approach and further investigations reveal interesting connections between the modal damping matrix, as well as the natural frequencies, and the accuracy of the approximation.

The major contributions of the article are the idea of designing linear systems for accurate proportional damping approximation using the energy gain of the error system and the algorithm developed to solve the resulting problem. This algorithm illustrates the effectiveness of combining simultaneous perturbation stochastic approximation and linear matrix inequalities techniques to solve structural design problems. Lastly, application to the design of tensegrity structures is important as well, especially in the context of the recent growth in tensegrity research [4,17].

2. Non-proportionally and proportionally damped systems

The system described by Eq. (1) can be cast into the modal form using the modal transformation, which is obtained from the mass and stiffness matrices as follows:

$$M = U_M \Lambda_M U_M^T, \quad U_M U_M^T = I, \quad \Lambda_M = Diag. > 0$$

$$\sqrt{\Lambda_M}^{-1} U_M^T K U_M \sqrt{\Lambda_M}^{-1} = U_K \Omega^2 U_K^T, \quad U_K U_K^T = I, \quad \Omega = Diag. > 0$$

$$U = U_M \sqrt{\Lambda_M}^{-1} U_K, \quad q = U q_m \tag{2}$$

where U is the modal matrix, q are the physical coordinates, and q_m are the modal ones. Transformation $q=Uq_m$ applied to Eq. (1) yields, in modal coordinates,

$$\ddot{q}_m + C_m \dot{q}_m + \Omega^2 q_m = U^T f \tag{3}$$

where $C_m=U^TCU$, $\Omega^2 = \text{diag}(\omega_l^2)$, and ω_l, $l = 1,\ldots,n$ are the natural frequencies given by

$$\det(K - \omega_l^2 M) = 0. \tag{4}$$

If the modal damping matrix, C_m, is diagonal then the system described by Eq. (1) is proportionally damped, the equations of motion given by Eq. (3) decouple, and they can be treated independently. This is extremely advantageous for complex physical systems described using many degrees of freedom (e.g. multi-body robots, structures), because proportionally damped models allow for the replacement of *large* non-proportionally damped, highly *coupled* models, with *smaller*, *decoupled* ones. A large system which decouples does not suffer from computational complexity problems because the decoupled equations can be solved independently. Moreover, the control design problem is tremendously simplified