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Improving frequency estimation performance for burst transmissions by optimising reference symbol distribution

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ABSTRACT

The theoretical lower bound on data-aided (DA) frequency estimation error for bursts transmissions containing reference symbols is known to decrease if the reference symbols are distributed throughout the burst. In practice, DA frequency estimators exhibit threshold behaviour where the performance rapidly degrades if the signal-to-noise ratio (SNR) is lower than a certain threshold value. Lowering the SNR threshold is often an important goal in the design of both estimators and signal formats.

This article examines DA frequency estimation for burst transmissions, specifically analysing the threshold behaviour for both regular and irregular reference symbol distributions. We demonstrate through analysis and simulation results that threshold behaviour can be determined from the frequency estimation likelihood function, particularly the magnitude and location of secondary likelihood peaks. We also demonstrate, for a specific test case, that significant improvements in threshold performance can be gained with irregular distribution of reference symbols, with little degradation in the frequency estimation error.

The methods described in this paper allow the signal designer to select a reference symbol distribution to optimise and trade-off estimation performance parameters such as estimation error, acquisition range, outlier probability and threshold behaviour.

1. Introduction

Burst communication systems that operate at low signal to noise ratios often include known reference symbols in the burst transmissions to aid signal acquisition and synchronisation at the receiver. By processing the received reference symbol sequence, the receiver can estimate offsets in symbol timing, frame timing, carrier phase and carrier frequency [1]. The benefit of simplifying receiver processing has traditionally resulted in consecutive placement of reference symbols within the burst, and this placement is also convenient for symbol timing recovery and frame synchronisation. However, the accuracy of carrier frequency estimation increases if the reference symbols are more widely distributed, as in pilot symbol assisted modulation (PSAM) [2].

In this paper, we analyse frequency estimation performance as a function of reference symbol distribution, assuming ideal matched filtering, symbol timing and frame timing. We consider two methods of distributing the reference symbols within the available window inside a burst:

Regular—where the reference symbols are grouped into equal length partitions which are then distributed within the burst at regular intervals.

Irregular—where each reference symbol is placed within the window independently of other reference symbols in a pseudo-random or non-deterministic manner.

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The estimation performance for each distribution method is analysed in terms of RMS estimation error and outlier probability as a function of SNR. The outlier probability also determines the SNR threshold for a particular reference symbol distribution. Both RMS estimation error and outlier probability are determined from the shape of the frequency estimation likelihood function, which in turn is determined by the distribution of the reference symbols. Careful selection of the reference symbol distribution can lead to significant gains in estimator performance in the threshold region.

This paper is organised as follows. Section 2 provides a brief review of data-aided (DA) frequency estimation, including performance bounds for estimation error an outlier probability. The frequency estimation performance bound for the regular reference symbol distribution is then derived in Section 3. The relationship between the shape of the frequency estimation likelihood function and the acquisition range and threshold performance of the DA estimator is also explained. Section 4 demonstrates the performance benefits that can be obtained from irregular reference symbol distributions. Finally, the findings of this paper are summarised in Section 5.

2. Data-aided frequency estimation

DA frequency estimation can be used to estimate the frequency offset present in a modulated signal if the signal contains reference symbols whose values are known a priori at the receiver. These reference symbols will typically be used in the receiver to perform frame and symbol timing synchronisation, but they can also be used for frequency estimation purposes [1].

Assuming frame synchronisation has occurred, the demodulator has knowledge of the position of reference symbols within the received signal, and the values of those reference symbols. For the purposes of synchronisation, non-reference symbol samples can be zeroed, and the modulation can be stripped from the received reference symbols x_k by multiplying them by the conjugate of the known symbol values u_k. We assume the reference symbols are of equal amplitude. In this case, the resulting frame of modulation-stripped reference symbol samples y_m represents samples of a complex sinusoid whose frequency represents the frequency offset of the received signal, plus additive noise:

$$y_m = \begin{cases} Ce^{j\omega_o m+\phi_o}+n_m, & m \in K \\ 0, & m \notin K \end{cases} \quad (1)$$

where m indexes the received symbols from $1..N$; C represents the received signal amplitude; K is the set of reference symbol locations; ω_o is the frequency offset, normalised to the frame symbol rate; ϕ_o is an arbitrary phase offset for the frame, and n_m represents uncorrelated, white Gaussian noise.

2.1. Cramer–Rao lower bound on frequency estimation error

For the case where N reference symbols are placed consecutively within the modulated signal, the maximum likelihood frequency estimate [3] is found by maximising the frequency estimation likelihood function

$$A(\omega) = \left| \sum_{m=0}^{N-1} y_m e^{-j\omega m} \right| \quad (2)$$

with respect to ω. The Cramer–Rao lower bound (CRLB) on the estimation error variance for this case [3] is given by

$$cr(\omega) = \frac{6}{\frac{E_S}{N_O}N(N^2-1)} \quad (3)$$

where E_S/N_O represents the signal to noise ratio,[1] E_S is the received signal energy per symbol and N_O is the noise power spectral density.

In the more general case, the N reference symbols need not be placed consecutively, but can be placed at arbitrary locations k_m within the M-symbol received signal frame, where

$$k_m \in \{0,1,2,\ldots,M-1\} \quad (4)$$

In this case, the frequency estimate is found by maximising the modified frequency estimation likelihood function [4] shown below

$$A(\omega) = \left| \sum_{m=0}^{N-1} y_{k_m} e^{-j\omega k_m} \right| \quad (5)$$

with respect to ω, and the estimation error CRLB [4] is given by

$$cr(\omega) = \frac{N/2}{\frac{E_S}{N_O}\left(N\sum_{m=0}^{N-1} k_m^2 - \left(\sum_{m=0}^{N-1} k_m\right)^2\right)} \quad (6)$$

The frequency estimation likelihood function (5) is a continuous function of frequency. A frequency sampled version of the likelihood function can be obtained via a discrete Fourier transform (DFT) of a frame of modulation-stripped reference symbol samples, with the non-reference symbols masked out by zeros. If the DFT resolution is high enough (at least 3 times oversampled), the exact frequency which maximises the likelihood function can be found with negligible error by interpolating the DFT output around the peak bin to locate the maximum [5].

2.2. Frequency estimation outlier probability and threshold performance

Maximum likelihood (ML) DA frequency estimators are efficient at high signal-to-noise ratios (SNR) [6], in that the estimation error variance achieved is that predicted by the theoretical bounds. However, these estimators often exhibit an SNR threshold, below which the estimation error variance increases significantly above the

[1] For the signal model presented in (1), since all symbols are of equal amplitude, the energy per symbol E_S is C^2/r_S, the signal power divided by the symbol rate. The estimation algorithm operates with one sample per symbol, so the signal bandwidth is the symbol rate r_S, and the noise power spectral density N_O is therefore σ_n^2/r_S, the noise variance divided by the symbol rate. E_S/N_O is therefore C^2/σ_n^2, the signal power divided by the noise power.