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Higher-order spectra for identification of nonlinear modal coupling

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ABSTRACT

Over the past four decades considerable work has been done in the area of power spectrum estimation. The information contained within the power spectrum relates to a signal's autocorrelation or 'second-order statistics'. The power spectrum provides a complete statistical description of a Gaussian process; however, a problem with this information is that it is phase blind. This problem is addressed if one turns to a system's frequency response function (FRF). The FRF graphs the magnitude and phase of the frequency response of a system; in order to do this it requires information regarding the frequency content of the input and output signals.

Situations arise in science and engineering whereby signal analysts are required to look beyond second-order statistics and analyse a signal's higher-order statistics (HOS). HOS or spectra give information on a signal's deviation from Gaussianity and consequently are a good indicator function for the presence of nonlinearity within a system.

One of the main problems in nonlinear system identification is that of high modal density. Many modelling schemes involve making some expansion of the nonlinear restoring force in terms of polynomial or other basis terms. If more than one degree-of-freedom is involved this becomes a multivariate problem and the number of candidate terms in the expansion grows explosively with the order of nonlinearity and the number of degrees-of-freedom.

This paper attempts to use HOS to detect and qualify nonlinear behaviour for a number of symmetrical and asymmetrical systems over a range of degrees-of-freedom. In doing so the paper also attempts to show that HOS are a more sensitive tool than the FRF in detecting nonlinearity. Furthermore, the object of this paper is to try and identify which modes couple in a nonlinear manner in order to reduce the number of candidate coupling terms, for a model, as much as possible. The bispectrum method has previously been applied to simple low-DOF systems with high symmetry and has been shown to work well in this limited case. The current paper will consider a model of a continuous wing–pylon model with reduced symmetry in order to assess the utility of the method in a more general situation, the analysis is also extended to assess the utility of the trispectrum.

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1. Introduction

For the past 40 years the use of higher-order statistics (HOS) has become a growing field of interest, [1]. HOS are used in the analysis and interpretation of signals from nonlinear systems. The development and utilisation of the fast Fourier transform (FFT), enables one to reveal information regarding the distribution of energy among a signal's frequency components. The power spectrum is regarded as the decomposition of a signal's power. For a zero-mean process, power is defined as the signal's mean-squared value. This concept is extended for HOS, HOS considers the higher-order products of the signal.

A zero-mean random process is described by its second-order statistics, i.e. the autocorrelation function and the power spectrum. These, however, are only a partial description of an arbitrary process [2].

Consider two discrete random processes. One is uniform white noise, the other is a Gaussian noise sequence. Both these processes have the same spectral characteristics and thus the same correlation structure, however their time-series differ. Whilst uniform noise has strict upper and lower bounds, its Gaussian counterpart does not. It is clear that second-order statistics cannot decipher between the two. A solution to this problem is the extension to HOS.

The HOS of primary concern is the third-order spectrum, the 'bispectrum'. This is the simplest of the HOS. The bispectrum provides information in cases when the random process has a 'skewed' distribution (skewness is a measure of asymmetry of data around the mean). As a result the bispectrum is particularly of use when the analyst wishes to detect quadratic (or other even powers of) nonlinearity. The next in the series of higher-order spectra is the fourth-order spectra, the trispectrum. The trispectrum is particularly useful when analysing a signal for cubic (or other odd powers of) nonlinearity.

The objective of this paper is to investigate the use of HOS in identifying nonlinear modal coupling as a means of selecting significant model terms in nonlinear system identification. Before proceeding, it is important to state clearly what is meant here by 'modal' and 'coupling' in the context of multi-degree-of-freedom (MDOF) nonlinear systems.

One begins with the MDOF linear system. In a standard notation, this is described by

$$[m]\{\ddot{y}\} + [c]\{\dot{y}\} + [k]\{y\} = \{x\} \tag{1}$$

where $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices, respectively. $\{x(t)\}$ is the input force and $\{y\}$ is the corresponding displacement response. Square brackets denote matrices and curly brackets denote vectors. If the number of degree-of-freedom are N, then the matrices are $N \times N$, etc. For the linear system, modal analysis is unambiguous [3]. The modes or *normal modes* or *modeshapes* $\{\psi\}_i$, $i = 1, \ldots, N$, of the system are the eigenvectors of the linear eigenvalue problem,

$$([m] - \lambda[k])\{\psi\} = 0 \tag{2}$$

and the natural frequencies are encoded in the eigenvalues. The modes have a number of interesting properties. One property is that the modeshapes represent the proportions of the displacements when the system is excited at the corresponding natural frequency (strictly only for the undamped system) and the various coordinates all pass through zero simultaneously and reach optima simultaneously. For the purposes of this paper, the most important property is that the modeshapes can be arranged in a matrix $[\Phi]$, which generates a linear transformation to a basis where the original N DOF system decouples in N SDOF systems. (Again, this is only true for undamped or proportionally damped systems [3].) In order to describe the system in the physical coordinates $\{y\}$, one needs to specify $n(n-1)+2$ parameters (again for proportional damping); however, in the *modal* or *generalised* coordinates $\{u\} = [\Phi]\{y\}$, one only needs $2(n+1)$, which could represent a considerable saving in effort if one wishes to estimate the parameters in order to identify the system.

For nonlinear systems, it is not possible to define normal modes which simultaneously have all the properties of the linear modeshapes. One has to make a decision as to which properties one wishes to preserve. The first definition of nonlinear normal modes elected to define them as the patterns of movement which had all coordinates moving in phase and passing through zero simultaneously; this is the approach of Rosenberg [4]. Since then there has been considerable work on the subject; a particularly elegant approach is the geometrical one of Shaw and Pierre who defined the normal modes in terms of invariant manifolds [5]. Good overviews of the field up to about 1997 can be found in the book [6] and review paper [7]. A recent paper with more up-to-date references and a good pedagogical treatment is [8]. All of these references discuss that fact that if a system is excited with a single frequency and one of the parameters of the excitation is varied smoothly, then bifurcations can generate new 'modes' which are not analytic continuations of the linear modes (i.e. the modes of the linear system which results from deleting all nonlinear terms). This situation is not discussed further in this paper as all the systems considered here will be subject to broadband random excitation and bifurcations are not a concern.

If one returns to the identification problem for nonlinear systems, broadband excitation is known to be the preferred mode of excitation [9]. In this paper, it will further be assumed that the excitation is Gaussian. The following discussion takes place in the context of the direct parameter estimation (DPE) method of system identification described in [10], although the remarks apply to any number of identification strategies. In general, the nonlinear system of interest will be assumed to have the n-DOF equation of motion,

$$[m]\{\ddot{y}\} + [c]\{\dot{y}\} + [k]\{y\} + \{f_{nl}(\{y\}, \{\dot{y}\})\} = \{x\} \tag{3}$$