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Theoretical analysis on the finite-support approximation for the mixing-phase FIR systems ☆

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ABSTRACT

The inverse system approximation using the finite impulse responses (FIR) and the corresponding model-order determination are essential to a broad area of science and technology utilizing signal processing. To the best of our knowledge, there exists no explicit formulation of the exact L_2 approximation error for the truncated inverse filters. The approach to determine the minimum inverse model-order subject to the maximum allowable L_2 approximation error is also in demand. In this paper, we present two L_2 approximation error measures and the two corresponding optimal finite-support approximates. Also, we derive the explicit L_2 approximation error functions with respect to roots, multiplicities and model orders for these two kinds of approximates. Then, we propose a new algorithm to determine the minimum total model order of the appropriate truncated inverse filter to achieve a specified L_2 approximation error. Our newly derived L_2 approximation error evaluation method can be employed for signal processing, telecommunication, control systems involving the inverse filtering in the future. Besides, our novel model-order determination algorithm can be utilized for efficient dynamic memory allocation in a wide variety of applications since such a minimum total model order is proportional to the memory usage for any inverse filter implementation.

1. Introduction

The filters can be used for a lot of engineering applications [1]. Among all kinds of filters, the inverse filter design leads to numerous applications in industrial practice such as rectification of the distorted signals due to the measurement instrumentation [2], pathological characteristics extraction for laryngeal diseases [3], multi-resolution coding for video technology [4] and signal processing systems involving whitening and deconvolution [5,6].

In [7], the authors first used a pre-emphasis filter to whiten the power spectrum of the speech signal and then performed linear predictive analysis on the whitened speech to obtain the vocal tract parameters. Then, the speech residual signal is obtained by the inverse filter. A residual echo-cancelation scheme for the hands-free telephony was proposed in [8] and it simply applied the inverse filter to whiten the residual echo and reduce both acoustic echoes and ambient noise. An inverse filter for performing higher-order whitening and estimating the parameters associated with a non-Gaussian ARMA ran-

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dom signal was carried out in [9]. Similar works in [10–14] also addressed the inverse filter techniques for pre-whitening signals.

On the other hand, the inverse filter can be applied to rectify the distortion of a signal due to the measurement or the system imperfections. The received signal waveforms were compensated using an inverse filter to improve both transmitted and received focuses and outperform the time-shift estimation for aberration justification [15]. To determine a scattering matrix from the measurements subject to different antenna transfer functions, an inverse filter was designed to compensate the antenna transfer functions [16]. Other signal processing systems adopting inverse filters for signal rectifications could be found in [17–21]. Similar to signal rectification or equalization, deconvolution techniques are also crucial in signal processing and communications, where they primarily involve the inverse filter design [22–24].

The approximation of an inverse filter can be achieved using the optimization techniques such as Wiener filtering [25] and Hankel-norm approximations [26,27]. For example, an approximate maximum likelihood (ML) decision feedback block equalizer (A-ML-DFBE) for doubly selective (frequency- and time-selective) fading channels was proposed in [28]. This scheme can deal with the tradeoff between the complexity and the performance by adjusting the forward filter length. In [29], the authors developed an adaptive algorithm to improve the error rate performance based on the relation between the error probability and the equalizer length (inverse filter) for M-ary pulse amplitude modulation (M-PAM). It has been shown that the proposed equalizer scheme leads to better convergence performance in terms of speed and smoothness, compared with the traditional approximate minimum bit-error rate algorithm. However, the existing literature have not presented any error function with respect to the chosen model order (filter length) for a stabilized inverse approximate of an arbitrary mixing-phase system.

Due to the stability problem and the hardware constraints on the implementation of an infinite-impulse-response (IIR) inverse filter, it is worthwhile to design an approximated (truncated) inverse filter using a finite impulse response (FIR) [30]. Therefore, in this paper, we study this finite-support inverse approximation problem for mixing-phase FIR systems. We investigate two kinds of optimal finite-support approximates based on two different inverse filter approximation error criteria. Then we derive the explicit L_2 approximation error functions with respect to filter characteristics (roots, multiplicities associated with the transfer function, model orders, filter coefficients associated with the impulse response) in accordance with these two schemes. It is noted that we do not discuss about the algorithms for adaptive inverse filtering in this paper. Instead, we focus on the mathematical framework on the finite-support inverse filter approximation and the corresponding model-order determination, both of which cannot be established explicitly by the adaptive filtering paradigm. Different from the work in [28], we can quantify the tradeoff between the implementation complexity and the performance by exploring the explicit relationship between the L_2 approximation error and the filter length (model order) in this paper.

Besides, there exists no explicit approach so far to determine the minimum length of an approximated inverse FIR filter subject to the maximum allowable L_2 approximation error. We call this the *model-order determination* problem. The model-order determination can assure the minimum required subject-signal-to-interference ratio, which can be used to assess the signal quality resulting from an inverse system. Traditional model-order determination schemes are based on the trial-and-error techniques (repeatedly evaluating the approximation errors and checking if it is sufficiently small whenever the model order is adjusted) and the model orders for the maximum-phase and minimum-phase modes have to be determined separately. There exists no algorithm to combine the two modes and jointly determine the minimum *total model-order* (the summation of the model orders for the causal and anti-causal parts) subject to a given L_2 approximation error constraint. Hence, in this paper, we would like to not only derive the exact L_2 approximation error function but also introduce a novel model-order determination algorithm which can directly achieve the minimum total model order. Our new technique can be used to dynamically allocate the memory size which corresponds to the model order of the truncated inverse filter.

The rest of this paper is organized as follows. In Section 2, we provide a thorough discussion on the finite-support approximation for the stabilized mixing-phase inverse filters. In Section 3, we derive the explicit L_2 approximation error function between the truncated inverse filter and the actual inverse. Our novel minimum total-model-order determination algorithm is presented in Section 4. One application, the equalizer design for telecommunication systems, will be addressed in Section 5. The numerical evaluation results for demonstrating the effects of the roots associated with the filter coefficients and the model-order selection on the telecommunication equalization performance are presented in Section 6. Finally, concluding remarks will be drawn in Section 7.

Notations: The sets of all integers and positive integers are denoted as $\mathbb{Z}$ and $\mathbb{N}$, respectively. The symbol $\equiv$ is used to represent a mathematical definition. A *sequence* is expressed as $\langle a\rangle$ and the corresponding index-i element is represented as $\langle a\rangle_i$ where $i\in\mathbb{Z}$ thereupon. A sequence can also be represented as $\langle a\rangle=(\ldots,\alpha_{-2},\alpha_{-1},\overline{\alpha_0},\alpha_1,\alpha_2,\ldots)$, where the overline denotes the index-0 element in the sequence. $\langle\mathbf{1}\rangle$ represents the Dirac-delta sequence, i.e., $\langle\mathbf{1}\rangle\equiv(\ldots,0,0,\overline{1},0,0,\ldots)$. The L_2-norm of a sequence $\langle a\rangle$, denoted as $\|\langle a\rangle\|_2$, is $\|\langle a\rangle\|_2\equiv\sum_i|\langle a\rangle_i|^2$. $Z(\langle a\rangle)$ represents the Z-transform of the sequence $\langle a\rangle$; $Z^{-1}(F(z))$ represents the inverse Z-transform of a z-function $F(z)$ if such a sequence exists. If k is an arbitrary non-negative integer and $A(z)=a_0+a_1z^{-1}+\cdots+a_kz^{-k}$, then we may define the degree of $A(z)$ as $\deg(A(z))=k$. $\binom{p}{q}$ is defined as the number of combination of q distinct objects chosen from p distinct objects.