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# Increasing the robustness of fault identification in rotor dynamics by means of M-estimators

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**Abstract**

One of the most common problems in rotor dynamics is the identification of faults and model-based methods are often used for this purpose. In some applications, the least-squares (LS) estimate is used to find out the position and the severity of impending faults on the basis of experimental vibration data of rotating machinery. Anyhow LS are not very robust with respect to possible outliers (noise and gross errors) in the experimental data and to inaccuracies in the model.

The introduction of weights in the LS algorithm has proven to be effective in increasing the robustness and successful experimental cases, both on test rigs and on real machines, are reported in literature. However, the arbitrary choice of the weights is normally based on operators' experience. In this paper, an improvement is presented by introducing a method that is robust in itself, the M-estimate, which allows defining automatically the weights. This method is general and can be applied in every problem of regression or estimation, not necessarily related to rotor dynamics.

The fundamental theoretical aspects are introduced in the first part, while several experimental test cases are presented by means of fault identification on a test rig and on a gas turbo generator in the second part of the paper. The obtained results highlight the increasing of the accuracy allowed by M-estimate.


*Keywords:* Robust estimation; M-estimator; Identification; Diagnostics; Rotor dynamics; Unbalance

## 1. Introduction

It is well known that least squares (LS) estimate can badly behave when the error distribution is not normal, particularly when the errors are heavy tailed [1]. One remedy is to remove influential and corrupting observations from the LS fit or to use weighted LS [2]. These operations should be normally performed by skilled operators, since the deletion or the attribution of weights is rather arbitrary.

The advantages of the implementation of a robust technique are that the experimental data do not need to be inspected in order to check for the presence of corrupting noise or gross errors.

A suitable approach is the use of robust regression [3], i.e. the employing of a fitting criterion that is not as vulnerable as LS to outlying data and it is able to indicate the possible inaccuracies in the model of the system.

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The paper proposes the implementation of the robust regression by using the M-estimators: the standard LS method tries to minimize the sum of the square residuals of every data. Outlying data give an effect so strong in the minimization that the estimated parameters are distorted. The M-estimators try to reduce the effects of the outliers by replacing the square of the errors by another function of the errors. Several functions of the errors have been proposed [1], but whatever is chosen, the implementation is made by using iteratively the re-weighted least squares (IRLS) algorithm. Practically, the algorithm recursively assigns weights to data in order to minimize the effect of outliers. Whilst M-estimate is used in many different fields after its introduction in statistics, such as image recognition (i.e. [4,1]), electrical power systems (i.e. [5]) or chemical engineering (i.e. [6]), there are no applications on mechanical systems. A possible explanation could be that the analytical methods developed in statistics, and applied to other fields, handle data that are composed of real numbers, while often experimental data of mechanical systems are vibrations that can be suitably represented by means of complex numbers, i.e. amplitude and phase. In this paper, the authors introduce the M-estimate applied for identification of mechanical systems, especially to rotor dynamics. A preliminary analysis [7] has been already performed to select the most suitable type of M-estimator to be applied to mechanical systems.

In order to show the effectiveness and the performances of the proposed method, the paper presents several experimental cases of balancing mass identification in a test rig and in an industrial machine, showing the increasing accuracy in the estimation of the unbalance.

## 2. General approach to model-based identification of fault in the frequency domain for rotor system

The model-based identification of faults in rotor systems is essentially a multiple inputs, multiple outputs (MIMO) inverse problem. Some applications in rotor dynamics have been presented in both the time domain [8] and the frequency domain. Multiple faults can be considered as inputs and the vibrations, measured in different planes along two orthogonal directions, are the multiple outputs. Previous papers of the same authors [9–11] have shown how to define the model of the system and the model of common faults in rotor dynamics by means of equivalent excitation systems.

With regard to the experimental data, *additional vibrations* are used. They are obtained by means of the difference between the faulty condition and the reference condition. Under the hypothesis of linearity of the system, which is satisfied in many cases of common faults in rotating machinery, the additional vibrations are caused by the developing faults only. Further discussion about this topic can be found in [12,13].

Anyhow, since many types of faults of rotating machinery have effect, and thus related symptoms are acting, on few harmonic components (as is well known in the literature starting from Sohre's chart), the harmonic balance approach is used. The following equations are obtained for each harmonic component, in which the equivalent excitation vector $\mathbf{F}_{f_n}$ has to be identified:

$$\left[-(n\Omega)^2[\mathbf{M}] + in\Omega[\mathbf{C}(\Omega)] + [\mathbf{K}]\right]\mathbf{X}_n = \mathbf{F}_{f_n}, \tag{1}$$

where $[\mathbf{M}]$ is the mass matrix, $[\mathbf{C}(\Omega)]$ is the complete damping matrix that includes also the gyroscopic matrix calculated at the speed $\Omega$, $[\mathbf{K}]$ is the stiffness matrix.

Because the system is considered as linear, the effect of $m$ faults developing simultaneously can be considered by means of the superposition of the effects for each harmonic component:

$$\mathbf{F}_{f_n} = \sum_{i=1}^{m} \mathbf{F}_{f_n}^{(i)}. \tag{2}$$

These are the multiple inputs (MI) of the system. Moreover, the $k$th fault acts on few degrees of freedom (d.o.f.'s) of the system; therefore the vector $\mathbf{F}_{f_n}^{(k)}$ is not a full-element vector, which is convenient to be represented by means of

$$\mathbf{F}_{f_n}^{(k)} = \left\{\mathbf{F}_L^{(k)}\right\}\theta^{(k)}(\Omega), \quad \theta^{(k)}(\Omega) \in \mathbb{C}, \tag{3}$$

where $\{\mathbf{F}_L^{(k)}\}$ is the localisation vector which has all null elements, except for the d.o.f.'s to which the exciting system is applied, and $\theta^{(k)}(\Omega)$ is a complex number representing the amplitude and the phase of the fault. Obviously, as many nodes are used for the model as much the location of the fault is accurate.