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Linear parameter estimation for multi-degree-of-freedom nonlinear systems using nonlinear output frequency-response functions

Z.K. Peng*, Z.Q. Lang, S.A. Billings

Department of Automatic Control and Systems Engineering, University of Sheffield, Mappin Street, Sheffield S1 3JD, UK

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Abstract

The Volterra series approach has been widely used for the analysis of nonlinear systems. Based on the Volterra series, a novel concept named nonlinear output frequency-response functions (NOFRFs) was proposed by the authors. This concept can be considered as an alternative extension of the classical frequency-response function for linear systems to the nonlinear case. In this study, based on the NOFRFs, a novel algorithm is developed to estimate the linear stiffness and damping parameters of multi-degree-of-freedom (MDOF) nonlinear systems. The validity of this NOFRF-based parameter estimation algorithm is demonstrated by numerical studies.

Keywords: Nonlinear system; Volterra series; Parameter estimation; MDOF system; Nonlinear vibration

1. Introduction

Various methods have been developed to estimate the stiffness and damping parameters for linear structures or machines. Most of these are based on modal analysis techniques, which were essentially derived from the frequency-response functions (FRFs) [1–5]. To tackle the problem with finite element model updating, Arruda and Santos [1] estimated the mechanical parameters via curve fitting for measured FRFs using a nonlinear least-squares method. Sunder and Ting [2] used the system parameter estimation method to detect the occurrence and location of damage on steel jacket offshore platforms. Also based on the FRFs, Hwang [3] put forward an identification method for stiffness and damping parameters of connections using test data for a structure attached to another structure via connections. Woodgate [4] studied the problem of identifying a positive semi-definite symmetric stiffness matrix for a stable elastic structure from measurements of its displacement in response to some set of static loads. Most recently, Živanović et al. [5] described a lively full-scale footbridge from its numerical modelling and dynamic testing. Their work is a successful application of the FRFs to system parameter estimation in practice.

*Corresponding author. Tel.: +44 77233 42361.
E-mail addresses: z.peng@sheffield.ac.uk (Z.K. Peng), z.lang@sheffield.ac.uk (Z.Q. Lang), s.billings@sheffield.ac.uk (S.A. Billings).

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Nomenclature

$x(t)$, $u(t)$ the output and input of the nonlinear system
$X(\mathrm{j}\omega)$, $U(\mathrm{j}\omega)$ the spectrum of the system output and input
$h_n(\tau_1,\ldots,\tau_n)$ the nth-order Volterra kernel
$H_n(\mathrm{j}\omega_1,\ldots,\mathrm{j}\omega_n)$ the nth-order GFRF
$G_n(\mathrm{j}\omega)$ the nth-order NOFRF
Ω_n the frequency components of the nth-order output of the system subjected to harmonic inputs
Ω the frequency components of the output of the system
$G_n^H(\mathrm{j}\omega)$ the nth-order NOFRF of the system subjected to harmonic inputs
M, K, C the system mass, damping and stiffness matrices
m_i, k_i, c_i the ith mass, damping and stiffness parameter
$S_{LS}(\Delta)$, $S_{LD}(\dot{\Delta})$ the restoring forces of the nonlinear spring and damper
r_i, w_i $(i = 1,\ldots,P)$ the nonlinearity-related parameters
Non-F the nonlinear force
$x_i(t)$, $X_i(\mathrm{j}\omega)$ the displacement and the output frequency response of the ith mass
$h_{(i,j)}(\tau_1,\ldots,\tau_j)$ the jth-order Volterra kernel associated to the ith mass
$G_{(i,l)}(\mathrm{j}\omega)$ the lth-order NOFRF associated to the ith mass
$\lambda_n^{i,i+1}(\mathrm{j}\omega)$ the ratio between the nth NOFRFs of the ith and $(i+1)$th masses
W the vector of the unknown parameters to be estimated
$\Gamma_{(L-1,Z)}(\mathrm{j}\omega)$ the term introduced by the nonlinear force *Non-F* for the Zth-order NOFRF.

However, there are certain types of qualitative behaviour, which cannot be produced by linear models [6], encountered in engineering, for example, the generation of harmonics and inter-modulation behaviours. In cases where these effects are dominant or significant nonlinear behaviours exist, nonlinear models are required to describe the system, and the linear FRFs are no longer suitable to investigate the system dynamics.

The Volterra series approach [7] is a powerful tool for the analysis of nonlinear systems, which extends the familiar concept of the convolution integral for linear systems to a series of multi-dimensional convolution integrals. The Fourier transforms of the Volterra kernels are known as the kernel transforms, higher-order frequency-response functions (HFRFs) [8], or generalised frequency-response functions (GFRFs), and these provide a convenient tool for analysing nonlinear systems in the frequency domain. If a differential equation or discrete-time model is available for a system, the GFRFs can be determined using the algorithms in [9–11]. The GFRFs can be regarded as the extension of the classical FRF of linear systems to the nonlinear case. So far only a few researchers have addressed the problem of nonlinear system parameter estimation for nonlinear systems using the GFRFs. Lee proposed a straightforward method to estimate the nonlinear system parameters using the GFRFs [12]. Khan and Vyas [13] employed the relationships between higher-order GFRFs and first-order GFRF to estimate the nonlinear parameters. Later, Chatterjee and Vyas [14] further developed this method by using a method of recursive iteration.

In engineering practice, for many mechanical and structural systems, more than one coordinate is needed to sufficiently describe the system dynamics. The result is a multi-degree-of-freedom (MDOF) model. In addition, there are considerable mechanical and structural systems that behave nonlinearly just because one or a few components within the system are nonlinear. One well-known example is beam structures [15] with breathing cracks, the global nonlinear behaviours of which are caused only by the cracked elements. Such nonlinear MDOF systems can be regarded as locally nonlinear MDOF systems. An important fact is that, for such nonlinear systems, the linear stiffness and damping are still the decisive characteristics, which mainly determine the system behaviour. Therefore, a knowledge of the linear stiffness and damping are still of great significance for understanding the whole system dynamical properties.

In this paper, a novel method is proposed to estimate the linear stiffness and damping parameters for locally nonlinear MDOF systems. The method is based on the concept of nonlinear output frequency-response functions (NOFRFs) [16], which was recently proposed by the authors and is an alternative extension of the