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Generalized mutual information approach to multichannel blind deconvolution

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Abstract

In this paper, we propose an approach to the multichannel blind deconvolution problem, based on the mutual information criterion and more generally on an appropriate system of estimating equations. Formulas for the quasi-Newton algorithm and the asymptotic covariance matrix of the estimator are provided. More interesting results have been obtained in the pure deconvolution case. By a clever parameterization of the deconvolution filter, the estimated parameters are asymptotically independent and explicit and simple formula for their variance are obtained. The quasi-Newton algorithm also becomes particularly simple. Simulation results showing the good performance of the methods are provided.

Keywords: Blind source separation; Convolutive mixture; Mutual information; Entropy; Estimating equation; Deconvolution; Contrast

1. Introduction

Blind source separation aims at separating sources from their mixtures, without relying on any specific knowledge other than their independence [1]. In the case of convolutive mixtures, however, this assumption alone can only permit to separate the sources up to a convolution, so that further assumptions may be introduced to eliminate this ambiguity. A common assumption is the temporal independence of the source signals, in this case the problem may be called multichannel blind deconvolution, since it reduces to the well-known blind deconvolution problem when there are only one source and one sensor. Specifically, K observed sequences $\{X_1(t)\}, \ldots, \{X_K(t)\}$ are related to K source sequences $\{S_1(t)\}, \ldots, \{S_K(t)\}$ (we assume here that there are the same number of sources as sensors) via linear convolutions

$$\mathbf{X}(t) = \sum_{u=-\infty}^{\infty} \mathbf{A}(u)\mathbf{S}(t-u), \tag{1}$$

where $\mathbf{X}(t) = [X_1(t) \ \cdots \ X_K(t)]^{\mathrm{T}}$ and $\mathbf{S}(t) = [S_1(t) \ \cdots \ S_K(t)]^{\mathrm{T}}$, $^{\mathrm{T}}$ denoting the transposition, and $\{\mathbf{A}(u)\}$ is a sequence of $K \times K$ of matrices. Sources and observations are assumed to be real. A model with noise may be considered, however, this work is specifically designed for the noiseless case and may not perform well in the presence of noise. The goal is to recover the sources from the observations,

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using only the available informations: the independence of the source sequences (blind separation) and the temporal independence of these sequences (blind deconvolution). Naturally, the recovered source sequences $\{Y_k(t)\}$ are obtained through a deconvolution matrix filter

$$\mathbf{Y}(t) = \sum_{u=-\infty}^{\infty} \mathbf{B}(u)\mathbf{X}(t-u), \tag{2}$$

where $\mathbf{Y}(t) = [Y_1(t) \ \cdots \ Y_K(t)]^{\mathrm{T}}$ and $\{\mathbf{B}(u)\}$ is a sequence of matrices to be determined. This is usually done by minimizing a criterion or solving a system of estimating equations.

There has been several proposals for criteria for blind separation of convolutive mixtures and multichannel blind deconvolution. Earlier criteria are mostly based on cross cumulants between the outputs $Y_k(t)$ [2–6]. More recently, frequency domain approach has attracted interest since the convolution becomes multiplication in the frequency domain via the Fourier transform. This gives rise to criteria based on (cross-) polyspectra [7–10]. However, criterion based on the mutual information has not received much attention. Such a criterion has been introduced in [11] but no algorithm for minimizing it has been investigated and no study on the asymptotic distribution of the estimator has been made. The paper [12] minimizes a cost function which looks similar to the mutual information criterion but is not quite the same, as the determinantal term is different. It should be noted that in the unichannel ($K = 1$) case, an algorithm based on the mutual information criterion has been introduced in [13]. The same criterion also has implicitly appeared earlier in [14] but in a simplified form as the entropy of the output, since the author prewhitens the signal so that the deconvolution filter transfer function would have unit amplitude. This form is similar to the infomax criterion, which was also proposed for the blind deconvolution [15].

This paper considers the mutual information approach to multichannel deconvolution. The multichannel case is much harder than the unichannel case since it involves both separation and deconvolution. We actually consider a more general class of estimators, which minimize some criterion generalizing the mutual information or are simply solutions of some system of estimating equations. Such generalization could reduce the computational cost, since the mutual information involves the unknown densities of the recovered sources, which must be estimated. However, the use of the mutual information (actually an estimated empirical version) should lead to an efficient (optimal) estimator as this criterion is closely related to the log likelihood [1]. This will be proved in the unichannel case. The same result might be proved in the multichannel case but due to the complexity of the calculations, we have not done so. This work is focused on the theoretical aspects of the problem. In particular, we derive formulas for the asymptotic covariance matrix of the estimator and for implementing the quasi-Newton algorithm for its computation. In the unichannel case, one can obtain a very simple quasi-Newton algorithm by working with the logarithm of the deconvolution filter and separating its real and imaginary parts. No such simplification is possible in the multichannel case and the quasi-Newton algorithm, as it stands, may be too complex to be useful.

Section 2 introduces the theoretical mutual information criterion and derives its gradient and Hessian. The next section considers a general estimation method, which includes the mutual information approach. Sections 5 and 4 provide formulas for the quasi-Newton algorithm and the asymptotic covariance matrix of the estimator. Section 6 specializes the results on the unichannel case and Section 7 provides some simulation examples. For ease of reading proofs of results are relegated to an Appendix.

The following notations will be used throughout:

- $\mathbf{I}$ denotes the identity matrix, $\mathbf{0}$ denotes the null matrix (or vector).
- For a matrix sequence $\{\mathbf{M}(u)\}$, $\{\mathbf{M}^\dagger(u)\}$ denotes its inverse (in the convolutive sense), defined by the condition

$$(\mathbf{M}^\dagger \star \mathbf{M})(u) \stackrel{\text{def}}{=} \sum_{v=-\infty}^{\infty} \mathbf{M}^\dagger(v)\mathbf{M}(u-v) = \begin{cases} \mathbf{I}, & u = 0, \\ \mathbf{0} & \text{otherwise.} \end{cases}$$

- For a matrix sequence $\{\mathbf{M}(u)\}$, its (discrete time) Fourier transform is denoted as $\mathbf{M}(\omega) = \sum_{u=-\infty}^{\infty} \mathbf{M}(u)\mathrm{e}^{-\mathrm{i}\omega u}$. The same symbol $\mathbf{M}$ has been reused for simplicity but the variable ω (Greek letter) in contrast with u (Roman letter) helps to avoid the confusion.
- $\mathrm{tr}(\mathbf{M})$ denotes the trace of the matrix $\mathbf{M}$.