Available online at www.sciencedirect.com

ScienceDirect

Signal Processing 87 (2007) 2197–2212

SIGNAL PROCESSING

www.elsevier.com/locate/sigpro

Fast RLS Fourier analyzers capable of accommodating frequency mismatch ☆

Yegui Xiao[a,*], Liying Ma[b], Rabab Kreidieh Ward[c]

[a]*Department of Management and Information Systems, Prefectural University of Hiroshima, 1-1-71 Ujina-Higashi, Minami-Ku, Hiroshima 734-8558, Japan*

[b]*Department of Applied Computer Science, Tokyo Polytechnic University, Atsugi, Kanagawa 243-0297, Japan*

[c]*Institute for Computing, Information and Cognitive Systems, University of British Columbia, Vancouver, BC, Canada V6T 1Z4*

Received 30 September 2006; received in revised form 6 February 2007; accepted 2 March 2007
Available online 13 March 2007

Abstract

Adaptive Fourier analyzers are used to estimate the discrete Fourier coefficients (DFC) of sine and cosine terms of noisy sinusoidal signals whose frequencies are usually assumed known a prior. The recursive least squares (RLS) Fourier analyzer provides excellent performance, but is computationally very intensive. In this paper, we first present four fast RLS (FRLS) algorithms based on the inherent characteristics of the DFC estimation problem. These FRLS algorithms show approximately the same performance and indicate estimation capabilities that are quite similar to those of the RLS, while requiring considerably less computational cost. Second, the performance of the proposed FRLS algorithms is analyzed in detail. Difference equations governing their dynamics as well as closed-form expressions for their steady-state mean square errors (MSE) are derived and compared with those of the LMS Fourier analyzer. Third, the RLS and four FRLS algorithms are modified by incorporating an adaptive scheme, to alleviate the influence of undesirable frequency mismatch (FM) on their performance. Extensive simulations as well as application to real noise signals are provided to demonstrate the relative performance capabilities of the RLS and four FRLS algorithms, the validity of analytical findings, and ability of the modified RLS and FRLS algorithms to mitigate the influence of the FM.

Keywords: Adaptive Fourier analysis; RLS; LMS; Performance analysis; Convergence properties; Frequency mismatch

1. Introduction

Adaptive Fourier analysis offers both efficient and effective solutions to estimation, enhancement and reconstruction of sinusoidal signals in noise. Some real-life application areas of the adaptive Fourier analysis are digital communications, power systems, control including active noise/vibration control, biomedical engineering, pitch detection in automated transcription, etc., where we are concerned with the analysis of a sinusoidal signal in additive noise [1,6–18]. The frequencies of the sinusoidal signal are arbitrary, and are usually known or estimated in advance. Moreover, the signal of interest is nonstationary for most of the time. The discrete Fourier transform (DFT) and its

☆A part of this work was presented at ISCAS2004 [26]. This work was supported in part by The Satake Research Foundation, Higashi-Hiroshima, Japan.

*Corresponding author. Tel.: +81 82 251 9731; fax: +81 82 251 9405.
E-mail address: xiao@pu-hiroshima.ac.jp (Y. Xiao).

0165-1684/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2007.03.003

variants [2–5] may be considered for the analysis of the signal. However, there are two major problems that make them basically awkward: (1) the signal frequencies are arbitrary and may not be an integer multiples of the fundamental frequency of the DFT, and (2) the signal is nonstationary due to its time-varying nature of amplitudes of frequency components, e.g. musical signals, etc., and it is difficult to find a window with proper length that fits the degree of nonstationarity of the signal.

To circumvent these difficulties with the DFT-type algorithms, many adaptive algorithms have been proposed. Some of them are the Kalman filtering based techniques, the recursive least squares (RLS) algorithm, the simplified RLS algorithm, the LMS-like algorithms, etc., see e.g. [4–18,21–26] and references therein. The RLS algorithm [24] presents excellent performance, but its computational requirements are much more intensive compared to the LMS-type algorithms. In this work, we are focused on the RLS algorithm. Essentially, the RLS algorithm is nothing but a direct extension of the RLS algorithm used in adaptive FIR filtering to the Fourier analysis problem. However, the inherent uniqueness of the discrete Fourier coefficients (DFC) estimation problem is not fully harnessed to reduce the computational burden involved.

In this work, we first present four (4) fast RLS (FRLS) algorithms by using of the unique characteristics of the estimation problem. Extensive simulations conducted for various scenarios reveal that the proposed algorithms present approximately the same convergence rate and steady-state properties as the RLS. However, their computational burden is considerably reduced. Many types of FRLS algorithms have been proposed in the context of adaptive FIR and IIR filtering by properly manipulating the input auto-correlation matrix [30]. In adaptive frequency estimation, the idea of dealing with frequency components one by one using cascaded and/or parallel-form notch filters has been applied [10]. Therefore, it is natural to attempt to estimate the DFCs of frequency components one by one [14,15]. The RLS algorithm for Fourier analysis may be easily simplified based on the same idea. The first FRLS (FRLS-I) algorithm [26] is a product directly derived from this idea. Unfortunately, no effort, to the best of our knowledge, has been made to develop algorithms that are faster than this algorithm. The insights used to derive the other three fast RLS algorithms are not difficult to figure out, but we have not found in literature any similar development based on signal decomposition or any other theory that leads to the proposed fast RLS-type algorithms.

Since three of the proposed FRLS algorithms can be treated as ones with scalar variable step size parameters, their performance analysis is tractable. Performance of the proposed FRLS algorithms is analyzed in detail. Difference equations governing their dynamics and closed-form expressions for their steady-state mean square errors (MSE) are derived and compared with those of the LMS Fourier analyzer. This analysis enriches our understanding of the behaviors of both the proposed FRLS and the conventional RLS algorithms, since they all perform similarly. The analytical results obtained are also useful in predicting performance of a system where certain FRLS algorithm is implemented.

The third issue of this work is the compensation for performance degradation due to the frequency mismatch (FM). In all the above-mentioned adaptive algorithms, including the newly proposed FRLS algorithms, the signal frequencies are provided in advance. However, the frequencies of the signal may show some differences from the ones given to the analysis algorithms. That is, an FM, large or small, may exist in real applications. The existence of FM was first mentioned by Glover [16]. For example, in automated transcription of electronic piano sounds, the frequencies of each note of a piano may be slightly different from the ones specified by the international standard, due to the variation of product quality [8,22]. In dual-tone multiple frequencies (DTMF) signaling, a maximum frequency tolerance or FM of 1.5% is allowed by the related international standards [19–21]. The frequency drift and magnitude variations of harmonics in power need to be estimated and/or compensated in real time for monitoring and maintaining the power quality [1,9]. In narrowband active noise control systems, signal frequencies derived from the speed sensor, i.e., tachometer, may be slightly different from the true ones of the primary signal due to the sensor error [27,28]. In order to enhance the applicability of the adaptive Fourier analysis algorithms, we have to take care of the FM in order to compensate for the performance degeneration. For the LMS-based Fourier analyzer, we have developed a scheme to fix the FM problem [25], but nothing has been done with the RLS algorithm. In the third part of this work, we modify the original and the proposed FRLS algorithms by