Mechanical Systems and Signal Processing 23 (2009) 2178–2191

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/jnlabr/ymssp

Effects of noise on transfer entropy estimation for damage detection

L.A. Overbey [1], M.D. Todd *

Department of Structural Engineering, University of California, San Diego, La Jolla, CA 92093-0085, USA

ARTICLE INFO

Article history:
Received 27 May 2008
Received in revised form
15 March 2009
Accepted 21 March 2009
Available online 10 April 2009

Keywords:
Transfer entropy
Information theory
Damage detection
Structural health monitoring

ABSTRACT

Recent research has shown that transfer entropy can be effectively employed as a feature for nonlinearity detection and linear damage identification. However, computation of transfer entropy requires the estimation of non-parametric one-, two-, and three-dimensional probability density functions. Therefore, small random perturbations caused by noise could lead to large variances in transfer entropy estimates. In this paper, we evaluate the effect of input and output noise on estimation of transfer entropy, and how noise, in turn, affects the capability of transfer entropy as a damage detector in a structural health monitoring (SHM) application. A damage index from the transfer entropy is computed from the response of a simulated multi-degree-of-freedom oscillator subject to linear and nonlinear stiffness changes in the presence of various noise influences. Damage indices are also evaluated for an experimental frame structure. Based on the computational study, we find that input noise lessens the sensitivity of the damage feature by diminishing the ability of the non-parametric density estimators to produce low variance transfer entropy estimations. Despite this reduced capability, an input that has no deterministic component can still detect a 25% stiffness loss in the computational example employed. Output noise has a greater impact on the feature's ability to accurately estimate the transfer entropy, such that a signal-to-noise threshold of approximately 30 dB leads to a greatly diminished ability to detect damage. Despite these noise effects, all damage cases tested on an experimental frame structure were detectable using the transfer entropy damage index.

1. Introduction

Structural health monitoring (SHM) is a rapidly growing research field that attempts to characterize a structural condition in real-time using sensor measurements and data analysis techniques. Many SHM-related approaches rely on dynamical measurements of an externally excited structure's response [1]. The response data are then mined for features that indirectly indicate the presence, location, and/or extent of damage to the structure, commonly by comparing a baseline system condition to the current, possibly damaged, condition. The computed features often focus on examining linear relationships within the data, whereby system inputs and outputs are recorded and fitted to a linear model in the time or frequency domains. These linear models are based on the assumption that the system can be decomposed into its eigenstate (normal modes and frequencies). Frequency-domain-based linear approaches include the use of operating shapes ("peak picking"), rational polynomial curve fitting [2], and nonlinear least-squares fitting [3]. The Eigensystem Realization Algorithm [4] and Complex Exponential Algorithm [5] are two of several linear time-domain methods [6,7].

* Corresponding author. Tel.: +1 858 5345951; fax: +1 858 5346373.
E-mail addresses: lucas.overbey@navy.mil (L.A. Overbey), mdt@ucsd.edu (M.D. Todd).
[1] Current address: SPAWAR Systems Center Atlantic, P.O. Box 190022, N. Charleston, SC 29419-9022. Tel.: +1 843 2183668.

0888-3270/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ymssp.2009.03.016

However, the manifestation of damage is often characterized by the presence of nonlinearity in a previously (primarily) linear structure. Rattling in a bolted connection or opening and closing of a crack are two such examples. Therefore, it is instructive to be able to include nonlinear components in a system identification-based feature algorithm. Research has incorporated higher-order frequency response functions to account for weak nonlinearities in systems [8,9]. However, this method still assumes a specific model of the correlations present within the system.

A more general system identification approach can be made by characterizing the coupling within a system using information theory-based algorithms. Information theory was originally established as a means of quantifying the fundamental limits required for compressing and communicating data reliably [10]. Measures of information include the information entropy, mutual information, and Kullback–Leibler divergence [11]. Nichols et al. [12,13] has adapted one such measure, the transfer entropy [14], for SHM-related applications and nonlinearity detection. Transfer entropy effectively quantifies the amount of information transfer between two measurements by utilizing statistical conditional relationships in the data. Past research [12,13] has included a time lag to examine the interrelationships between processes at various time scales. More recently, Overbey and Todd [15] proposed a modified version of the transfer entropy where specific time lag choices can improve sensitivity. However, this work was conducted on a model-based estimation of the transfer entropy, focusing purely on linear systems excited by random Gaussian processes.

However, if there is nonlinearity in the system, or if the system is excited by other means, a full estimation approach is required [13]. Because transfer entropy inherently compares conditional statistical relationships, its computation from a time series requires estimation of one-, two-, and three-dimensional probability density functions (pdfs). These estimates can be quite computationally intensive, requiring either a large number of points or low noise variance in order to get accurate and repeatable measures of the transfer entropy. Hence, it is important to establish what the limitations of this feature are in terms of estimation and how different noise influences affect these limitations.

This work resolves these limits using both empirical and analytical means. We evaluate the influence of noise on transfer entropy estimates for a dynamic structural model. A generalized, non-parametric estimation is employed here for the first time on the modified transfer entropy introduced by Overbey and Todd [15]. Moreover, we thoroughly analyze the effects of noise on these estimations, and in turn, the capability of transfer entropy as a damage detection feature.

First, we look at the case of a linear structural system subject to both linear and nonlinear stiffness changes to represent damage. The effects of both input and output noise on estimation and damage detection capability are examined. We consider input noise as any variability that is introduced *within the active forcing* that is applied to a structure. Output noise is any random component introduced in the *response of the structure*, such that it can be modeled by additive noise to the measurement(s). After the study of these noise-induced effects, an experimental frame structure is employed to validate the transfer entropy estimation procedure in a real-life application. All of these systems are analyzed within the context of an SHM application, where transfer entropy is employed as the damage detection feature.

2. Transfer entropy

Transfer entropy was originally formulated as a method to explicitly account for underlying dynamics in systems where information can flow between processes [14]. We can begin by analyzing processes from the perspective of their relationship to a traditional Markov series. Defining a kth-order Markov process x such that its dynamics are conditional only on previous values of x up to a time lag k, the transition probability of said process can be written as $p(x(n+1)|x(n)^{(k)}) = p(x(n+1)|x(n),x(n-1),\ldots,x(n-k+1)) = p(x(n+1)|x(n)^{(k)}) = p(x(1)|x^{(k)})$, where n is dropped in the last expression for notational convenience. However, if x is also influenced by another process y, we can introduce a second transition probability such that x relies on the previous time histories of x and y: $p(x(1)|x^{(k)},y^{(l)})$, where $y^{(l)} = y(n),y(n-1),\ldots,y(n-l+1)$. From dynamical interdependence, the transfer entropy can be expressed as

$$T_{y\to x}(x(1)|x^{(k)},y^{(l)}) = \int\int\int p(x(1)|x^{(k)},y^{(l)})\log_2\left(\frac{p(x(1)|x^{(k)},y^{(l)})}{p(x(1)|x^{(k)})}\right)dx(1)dx^{(k)}\,dy^{(l)}. \tag{1}$$

In essence, transfer entropy quantifies the influence of y on the dynamics of x, independent of the influence of x on itself. A purely data-based formulation of the transfer entropy would require estimation of multi-dimensional probability densities, where the dimension is determined by the assumed Markov orders k and l. Therefore, it is instructive to reduce the order of the processes to $k = l = 1$ for computational simplicity, and introduce a time delay to assess the effects of pertinent time scales on the system dynamics. Previous research [15] has introduced a lag τ in y such that the transfer entropy equation can be written as

$$T_{y\to x}(x|x(\tau),y(\tau)) = \int\int\int p(x|x(\tau),y(\tau))\log_2\left(\frac{p(x|x(\tau),y(\tau))}{p(x|x(\tau))}\right)dx\,dx(\tau)dy(\tau). \tag{2}$$

where $x(1)$ was replaced with x for notational simplicity.