Available at
www.ElsevierComputerScience.com
POWERED BY SCIENCE DIRECT
Signal Processing 86 (2006) 572–588
SIGNAL PROCESSING
www.elsevier.com/locate/sigpro

Algorithms for simultaneous sparse approximation. Part I: Greedy pursuit☆

Joel A. Tropp*, Anna C. Gilbert, Martin J. Strauss

Department of Mathematics, The University of Michigan, 2074 East Hall, Ann Arbor, MI 48109-1109, USA

Received 10 November 2004; received in revised form 17 March 2005; accepted 24 May 2005
Available online 15 August 2005

Abstract

A simultaneous sparse approximation problem requests a good approximation of several input signals at once using different linear combinations of the same elementary signals. At the same time, the problem balances the error in approximation against the total number of elementary signals that participate. These elementary signals typically model coherent structures in the input signals, and they are chosen from a large, linearly dependent collection.

The first part of this paper proposes a greedy pursuit algorithm, called simultaneous orthogonal matching pursuit (S-OMP), for simultaneous sparse approximation. Then it presents some numerical experiments that demonstrate how a sparse model for the input signals can be identified more reliably given several input signals. Afterward, the paper proves that the S-OMP algorithm can compute provably good solutions to several simultaneous sparse approximation problems.

The second part of the paper develops another algorithmic approach called convex relaxation, and it provides theoretical results on the performance of convex relaxation for simultaneous sparse approximation.
© 2005 Elsevier B.V. All rights reserved.

Keywords: Greedy algorithms; Orthogonal matching pursuit; Multiple measurement vectors; Simultaneous sparse approximation; Subset selection

1. Introduction

In recent years, the signal processing community has lavished attention on the class of simple sparse approximation problems. These problems have two facets:

(1) A signal vector is approximated using a linear combination of elementary signals, which are drawn from a fixed collection. In modern problems, this collection is often linearly dependent and large.
(2) The problem seeks a compromise between the approximation error (usually measured with

☆This research has been supported by NSF DMS 0354600.
*Corresponding author. Tel.: +1 734 9369923; fax: +1 734 7630937.
E-mail addresses: jtropp@umich.edu (J.A. Tropp), annacg@umich.edu (A.C. Gilbert), martinjs@umich.edu (M.J. Strauss).

0165-1684/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.sigpro.2005.05.030

Euclidean distance) and the number of elementary signals that participate in the linear combination. The goal is to identify a good approximation involving few elementary signals—a *sparse* approximation.

Simple sparse approximation problems originally arose in the study of linear regression. In this setting, we wish to approximate a data vector using a linear combination of regressors, but we must control the number of regressors to avoid fitting noise in the data. Statisticians developed many of the numerical algorithms that are used for solving simple sparse approximation problems [1].

One striking generalization of simple sparse approximation has garnered little attention in the literature. Consider the following scenario. Suppose that we have several observations of a signal that has a sparse representation. Each view is contaminated with noise, which need not be statistically independent. It seems clear that we should be able to use the additional information to produce a superior estimate of the underlying signal. This intuition suggests that we study *simultaneous sparse approximation*:

Given several input signals, approximate all these signals at once using different linear combinations of the same elementary signals, while balancing the error in approximating the data against the total number of elementary signals that are used.

Simultaneous sparse approximation problems arise in several specific domains. For example, Rao and his colleagues have considered applications to magnetoencephalography [2] and to the equalization of sparse communications channels [3]. Gribonval has developed applications to blind source separation [4]. Malioutov et al. [5,6] have shown that source localization using a linear array of sensors can be posed as a simultaneous sparse approximation problem [5,6]. It is easy to imagine many other applications in statistics, wireless communications, and machine learning.

1.1. Contributions

This work examines simultaneous sparse approximation from the practical and the theoretical point of view.

In the first part of the paper, we propose a greedy algorithm that generalizes the familiar orthogonal matching pursuit procedure, which was developed for simple sparse approximation [7,8]. At each iteration, a greedy pursuit makes the best local improvement to the current approximations in hope of obtaining a good overall solution. The same algorithm has been developed independently in [9,10].

Then, we summarize some numerical experiments using this greedy algorithm. These experiments confirm our intuition that having multiple observations of a sparse signal can improve our ability to identify the underlying sparse representation. They also give a measure of how the algorithm's performance depends on the number of input signals, the level of sparsity, and the signal-to-noise ratio (SNR).

Afterward, we prove that the greedy algorithm can calculate good solutions to simultaneous sparse approximation problems. Moreover, if we have some basic information about the signals, this information can be used to enhance the performance of the algorithm. Our proofs require that the collection of elementary signals possess a geometric property called *incoherence*. Roughly, incoherence means the elementary signals are weakly correlated with each other. The theoretical arguments build on work in [11–13].

In the second part of the paper, we develop a more sophisticated numerical method for simultaneous sparse approximation based on convex relaxation. Convex relaxation replaces the difficult simultaneous sparse approximation problem by a convex optimization problem, which can be solved in polynomial time with standard mathematical programming software. Using a variation of the argument in [14], we prove that convex programming yields good solutions to simultaneous sparse approximation problems, even in the presence of noise.

Our analysis of these two algorithmic methods for simultaneous sparse approximation yields the

Download English Version:

https://daneshyari.com/en/article/562039

Download Persian Version:

https://daneshyari.com/article/562039

Daneshyari.com