Available online at www.sciencedirect.com
ScienceDirect
Mechanical Systems and Signal Processing 22 (2008) 295–314
Mechanical Systems and Signal Processing
www.elsevier.com/locate/jnlabr/ymssp

ARMA modelled time-series classification for structural health monitoring of civil infrastructure

E. Peter Carden*, James M.W. Brownjohn

Department of Civil and Structural Engineering, University of Sheffield, Sir Frederick Mappin Building, Sheffield S1 3JD, UK

Received 20 July 2006; received in revised form 16 July 2007; accepted 18 July 2007
Available online 22 July 2007

Abstract

Structural health monitoring (SHM) is the subject of a great deal of ongoing research leading to the capability that reliable remote monitoring of civil infrastructure would allow a shift from schedule-based to condition-based maintenance strategies. The first stage in such a system would be the indication of an extraordinary change in the structure's behaviour.

A statistical classification algorithm is presented here which is based on analysis of a structure's response in the time domain. The time-series responses are fitted with Autoregressive Moving Average (ARMA) models and the ARMA coefficients are fed to the classifier. The classifier is capable of learning in an unsupervised manner and of forming new classes when the structural response exhibits change.

The approach is demonstrated with experimental data from the IASC–ASCE benchmark four-storey frame structure, the Z24 bridge and the Malaysia–Singapore Second Link bridge. The classifier is found to be capable of identifying structural change in all cases and of forming distinct classes corresponding to different structural states in most cases.

Keywords: Structural health monitoring; Time-series modelling; ARMA models

1. Introduction

The last two decades have seen a great deal of research and publication in the field of structural health monitoring (SHM) and there has been a proliferation of SHM paradigms put forward [1–3], with long-term monitoring systems implemented on bridges in Europe, the United States and Asia [4–6]. This has provided a great deal of practical experience and knowledge and helped SHM reach a greater level of maturity.

Despite this level of research, a robust method of indicating an adverse condition of a structure in service has yet to be demonstrated and widely implemented. This level of SHM is generally referred to as level 1. The classification algorithm proposed in this paper is primarily aimed at this level but can also provide a heuristic guide to the condition of the structure depending on the level of prior knowledge.

*Corresponding author. Tel.: +44 114 222 5745; fax: +44 114 222 5700.
E-mail address: peter.carden@sheffield.ac.uk (E. Peter Carden).

0888-3270/$ - see front matter
doi:10.1016/j.ymssp.2007.07.003

Modelling of time histories is common in many disciplines and has been used to study various phenomena from sunspot activities to stock market indexes. The literature on the subject is extensive. A classic treatment is provided by Box et al. [7] while a more system analysis focused approach, and perhaps a more directly useful primer to the vibration engineer, is given by Pandit and Wu [8].

It is possible to build linear stochastic models of structural time response histories using Autoregressive Moving Average (ARMA) models if it is supposed that the structure is randomly excited. An ARMA model of order (p, q) is defined

$$\phi(B)z_t = \theta(B)a_t, \tag{1}$$

where z_t is the time history response of the structure. a_t is the series of Gaussian distributed random shocks exciting the structure. $\phi(B)$ is the AR function of order p and $\theta(B)$ is the MA function of order q defined, respectively, as

$$\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p, \tag{2}$$

$$\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q, \tag{3}$$

B is the backward shift operator: $B^m z_t = z_{t-m}$.

The AR parameters are directly related to the system poles, λ_i, $i = 1, 2, \ldots, n$ through the following equation [8]. For the case of dynamic structural response involving a number of vibration modes, n is twice the number of system modes excited and each mode is represented by a complex conjugated pair of poles

$$\phi_l = (-1)^{l+1} \sum_{i_1, i_2, \ldots i_l = 1,\ i_1 < i_2 < \cdots < i_l}^{n} \lambda_{i_1} \lambda_{i_2} \ldots \lambda_{i_l}. \tag{4}$$

Each pair of poles is related to the natural frequency, ω_j, and damping ratio, ζ_j, of a mode through the following equation, where $i = 2j$:

$$\lambda_i, \lambda_{i-1} = \mathrm{e}^{\Delta(-\zeta\omega_j \pm \omega_j \sqrt{\zeta_j^2 - 1})}, \tag{5}$$

where λ_{i-1} is the complex conjugate of λ_i. Δ is the sampling time interval.

Thus, the natural frequency and damping ratio may be determined by examining only the AR parameters. This has led to the moving-average parameters generally receiving less attention in the system identification literature despite their importance for modelling. For example, Fig. 1 shows the power spectral densities of single-degree-of-freedom (SDOF) systems modelled as $ARMA(2, 1)$ systems. The analytical link between an SDOF and an $ARMA(2, 1)$ model is expounded in Appendix A. In Fig. 1, the AR parameters are held constant while the MA parameter is varied over a range of 0.5–2.5. The greater power in the signal with the MA parameter equal to 0.5 compared with 1.0 is not an error but rather results from the non-linear relationship between the MA parameter and the power in the mode. It should be noted that changing the MA

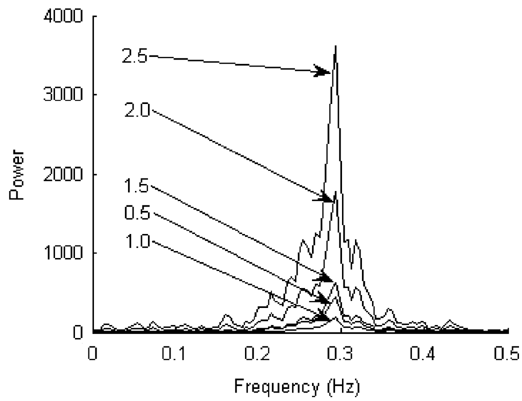

Fig. 1. Auto-power spectral densities of SDOF systems modelled with varying values of the MA parameter while maintaining the AR parameter values.