Signal Processing 123 (2016) 64–74

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

The construction of measurement matrices based on block weighing matrix in compressed sensing ☆

Hui Zhao *, Hao Ye, Ruyan Wang

School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

ARTICLE INFO

Article history:
Received 10 September 2015
Received in revised form
14 December 2015
Accepted 31 December 2015
Available online 11 January 2016

Keywords:
Compressed sensing
Block weighing matrix
Randomizer
Chaos-based permutation

ABSTRACT

In this paper, we propose a new structured measurement matrix for practical compressed sensing based on block weighing matrix, called partial Random Block Weighing Matrix (pRBWM). The proposed pRBWM is universal with a variety of sparse signals and provides high reconstruction performance simultaneously. In addition, with the sparse and circulant block structure, these new measurement matrices feature low-memory requirement and low computational complexity in reconstruction. Moreover, it can be more easily implemented in hardware thanks to its sample elements and the application of Chaos-based permutation operator in construction of pRBWM. Simulation results demonstrate that the proposed pRBWM performs comparably to, or even better than completely random matrices and many other structured matrices. And the proposed pRBWM forms a high balance between reconstruction performance,storage and computational complexity and hardware implementation.

1. Introduction

According to the well-known Nyquist sampling theorem in theory of digital signal processing, the sampling rate must be no less than twice the highest frequency of signal which always makes a high pressure on sampling and storage of practical sampling system. Nowadays, compressed sensing (CS) [1,2] has brought a revolution because of the breakthrough of low speed sampling of digital signal, thus attracted a lot of interests.

Consider a length-N signal $\boldsymbol{f}$ and sample it by a linear system, the data acquisition process in CS is described as:

$$\boldsymbol{y} = \boldsymbol{\Phi f}, \tag{1}$$

where $\boldsymbol{\Phi}$ is a $M \times N (M \ll N)$ measurement matrix, $\boldsymbol{y}$ represents a sampled vector with length M. The theory of CS implies that $\boldsymbol{f}$ can be faithfully recovered from only few measurements if signal $\boldsymbol{f}$ is K-sparse (or compressible) and $\boldsymbol{\Phi}$ satisfies some efficient conditions for exact reconstruction although the fact that the dimension of $\boldsymbol{y}$ is far below the dimension of original signal $\boldsymbol{f}$. Usually, natural signal is always not sparse itself but can be sparsified by some orthogonal transformations. Suppose that $\boldsymbol{f}$ can be well approximated by $K (K < M)$ non-zero coherences on a transform basis, i.e.

$$\boldsymbol{f} = \boldsymbol{\Psi\theta}, \tag{2}$$

where $\boldsymbol{\Psi} \in \mathbb{R}^{N \times N}$ is called as sparse basis matrix, $\boldsymbol{\theta} \in \mathbb{R}^{N}$ is the transform coefficient vector whose energy is mainly concentrated on at most K non-zero entries. And then the more general model of CS becomes

$$\boldsymbol{y} = \boldsymbol{\Phi f} = \boldsymbol{\Phi\Psi\theta} = \boldsymbol{\Theta\theta}, \tag{3}$$

where $\boldsymbol{\Phi\Psi} = \boldsymbol{\Theta}$ is called sensing matrix to distinguish $\boldsymbol{\Phi}$. To faithfully reconstruct $\boldsymbol{f}$ (or $\boldsymbol{\theta}$) from (3), sensing matrix

☆ Project supported by the National Nature Science Foundation of China (Nos. 61271261 and 61501075) and the Natural Science Foundation Project of Chongqing (CSTC2011jjjq40002 and KJ1400419) and Youth Science Foundation of Chongqing University of Posts and Telecommunications (A2015-59).

* Corresponding author.
E-mail address: zhaohui@cqupt.edu.cn (H. Zhao).

http://dx.doi.org/10.1016/j.sigpro.2015.12.016

$\boldsymbol{\Phi\Psi} = \boldsymbol{\Theta}$ must satisfy some strict criterions of which the most famous is the restricted isometry property (RIP) [3].

Definition 1. (RIP): An $M \times N$ matrix $\boldsymbol{\Theta}$ is said to satisfy the RIP with parameters $(K, \delta)(\delta \in (0,1))$ if

$$(1-\delta)\|\boldsymbol{\theta}\|_2^2 \le \|\boldsymbol{\Theta\theta}\|_2^2 \le (1+\delta)\|\boldsymbol{\theta}\|_2^2, \quad \text{for all } \boldsymbol{\theta} \in \boldsymbol{\Gamma}, \tag{4}$$

where $\boldsymbol{\Gamma}$ represents the set of all length-N vectors with K non-zero coefficients.

The RIP criterion is a sufficient condition for CS exact reconstruction, however it is hard to verify the RIP of a matrix, and thus the incoherence between the measurement matrix and sparse basis matrix has been proposed to guide the construction of CS measurement matrix in practice. The incoherence between the two matrices can be mathematically quantified by the mutual coherence coefficient [4]. The mutual coherence of an $N \times N$ orthonormal matrix $\boldsymbol{\Phi}$ and another $N \times N$ orthonormal matrix $\boldsymbol{\Psi}$ is defined as the largest absolute magnitude among the entries in $\boldsymbol{\Theta} = \boldsymbol{\Phi\Psi}$, i.e.

$$\mu(\boldsymbol{\Phi}, \boldsymbol{\Psi}) = \max_{1 \le i,j \le N} \left| \left\langle \boldsymbol{\phi}_i^T, \boldsymbol{\psi}_j \right\rangle \right|, \tag{5}$$

where $\boldsymbol{\phi}_i$ are rows of $\boldsymbol{\Phi}$ and $\boldsymbol{\psi}_j$ are columns of $\boldsymbol{\Psi}$ respectively. Because $\boldsymbol{\Phi}$ and $\boldsymbol{\Psi}$ are orthonormal, it is easy to know $1/\sqrt{N} \le \mu \le 1$. If μ approximates its minimum value of $1/\sqrt{N}$, then $\boldsymbol{\Phi}$ is viewed as incoherent with $\boldsymbol{\Psi}$ so that the sensing matrix $\boldsymbol{\Theta} = \boldsymbol{\Phi\Psi}$ will satisfy RIP with high probability.

At present, many random matrices have been proved to satisfy RIP with high probability, such as Gaussian and Bernoulli matrices [5,6]. It can be shown that random measurement matrix approaches the optimal sensing performance which requires least measurements for exact reconstruction. Moreover, random matrix is incoherent with any orthogonal sparse basis so that it can be universally used in variety of sparse transform domains. However, fully random matrices are difficult to implement in the hardware and require large memory and high computing complexity in reconstruction. Recently, researchers have developed some measurement matrices with low complexity by different novel methods. For example, binary sparse measurement matrix [7] with low complexity and near optimal performance has been developed by exploiting combinatorial algorithms. In addition, combing edge detection operator with completely random sampling operator, mixed adaptive-random (MAR) matrix [8] has been proposed for high-quality image restoration. However, these matrices are designed for specific signals.

Partial orthogonal matrix is another class of structured measurement matrices, such as partial Fourier transform matrix [5] and partial Hadamard transform matrix [9], which are generally constructed by randomly selecting rows of orthogonal matrices. Although the advantages of fast computing and friendship to hardware, these partial orthogonal matrices can only achieve optimal performance when signal is sparse itself in time domain while the performance may rapidly degenerate if the signal is sparse or compressible in some transform domains because it is difficult to guarantee that these measurement matrices are incoherent with the transform matrices (sparse basis matrices). To keep the good structure features of partial orthogonal matrices and make them suite for more sparse signals, several structured random matrices have been proposed. Among them, scrambled Fourier ensemble (SFE) [10] and scrambled block Hadamard ensemble (SBHE) [11] have applied the scrambling operator to make them incoherent with more orthonormal sparse basis matrices. Combing with completely random matrices and partial orthogonal sampling system, [12] has proposed the Structurally Random Matrices (SRM) in which SFE and SBHE can be viewed as the special cases. SRM can make a balance between reconstruction performance and computing, storage complexity, however it still have some drawbacks. Firstly, when SRM is dense (like SFE), the superiority of fast computing and small storage requirement will lose although its optimal reconstruction performance. Secondly, the reconstruction precision will fall if SRM is sparse especially when the signal is sparse in sparse or nonuniform basis. Moreover, the fully random permutation operator in SRM is always hard to hardware implementation [11].

In this paper we propose partial Random Block Weighing Matrix (pRBWM) for practical CS based on block weighing matrix (BWM) [13]. More precisely, this matrix is obtained by selecting rows uniformly at random from a matrix $\boldsymbol{\Phi} = \boldsymbol{FR}$, where $\boldsymbol{F}$ is a BWM with circulant block structure and $\boldsymbol{R}$ is the per-random operator from SRM framework. Moreover, to make pRBWM easy to be implemented in hardware, we introduce the Chaos-based permutation (CP) operator to replace the fully random permutation operator. The proposed pRBWM is universal with a variety of sparse signals and can achieve high reconstruction performance simultaneously from the theoretical perspective. In addition, from the practical perspective, pRBWM can be more easily implemented in hardware because of the excellent structure of BWM and the deterministic CP operator. Moreover, although proposed pRBWM is mainly developed on the SRM framework, it overcomes the aforementioned drawbacks of SRM in some degree.

The remainder of this paper is organized as follows. In Section 2 we briefly introduce the knowledge about BWM and the construction of circulant BWM. In Section 3, we propose pRBWM based on BWM and analyze their performance for CS exact reconstruction. In addition, the CP operator is proposed for practical considerations. Section 4 reports the simulation results followed by conclusions in Section 5.

2. The construction of block weighing matrix

Before presenting the proposed measurement matrices, we firstly introduce some knowledge about the weighing matrix (WM) and BWM, more details can be referenced in papers [13–16].