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Performance prediction of subspace-based adaptive detectors with signal mismatch ☆

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ABSTRACT

For the problem of subspace signal detection, three adaptive detectors have been proposed in the past, namely, the subspace-based generalized likelihood ratio test, subspace-based adaptive matched filter, and adaptive subspace detector. In this paper we analyze their performance in the mismatch case, where the actual signal does not exactly lie in the nominal signal subspace. We derive their statistical distributions, and then obtain analytical expressions for the probabilities of detection and false alarm. It is shown that the signal mismatch has a significant effect on their detection performance through a quantity, which can be taken as a measure of the "distance" between the actual signal and the nominal signal subspace. These results extend the existing theory for rank-one signal detection in the mismatch case.

1. Introduction

The problem of detecting a multichannel signal in unknown Gaussian noise arises in many areas, especially in radar systems [1]. The most famous adaptive detectors are Kelly's generalized likelihood ratio test (KGLRT) [2], adaptive matched filter (AMF) [3], and adaptive coherence estimator (ACE) [4]. For these three detectors, the signal has a precisely known steering vector. However, in practical applications, the actual signal steering vector may not be aligned with the presumed one, due to violation of underlying assumptions on the sensor array, environments, or sources [5]. A widely used method to design robust detectors is adopting a subspace model by assuming that the actual signal lies in a carefully-designed subspace, but with unknown coordinates [6]. In [7], the actual signal steering vector is assumed to lie in a given subspace. According to the generalized likelihood ratio test (GLRT) criterion, the authors in [7] propose the subspace-based GLRT (SGLRT), which is a generalization of the KGLRT. The SGLRT is independently derived in [8] in the context of polarimetric target detection. The corresponding two-step GLRT is first proposed in [9] when the dimension of the subspace is equal to two, and then it is generalized to the general-rank case in [10]. Essentially, it is the subspace version of the AMF. Hence, we call it the subspace-based AMF (SAMF). In addition, the subspace generalization of the ACE is dealt with in [11], referred to as the adaptive subspace detector (ASD) therein.

It is worth noting that although the subspace model is more robust to the signal mismatch, it cannot be guaranteed that the actual signal completely lies in the preassigned signal subspace. When a signal does not lie in the presumed subspace, we denote this phenomenon as subspace signal mismatch. There are some preliminary studies on signal detection in the presence of subspace signal mismatch. In [12,13] two robust detectors are designed for point-like targets when

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subspace signal mismatch occurs. In [14] the generalized adaptive direction detector (GADD) is proposed for the distributed target detection, which is more robust to subspace signal mismatch than other relative detectors. Notice that many papers investigate the statistical performance of subspace detectors in the absence of subspace signal mismatch, e.g., [7–10,15,16]. However, to the best of our knowledge, performance analysis of subspace detectors has not been studied for the case of subspace signal mismatch.

In this paper, we derive the statistical distributions of the SGLRT, SAMF, and ASD in the presence of subspace signal mismatch, according to which we obtain analytical expressions for the probabilities of detection (PDs) and false alarm (PFAs) of these detectors. Remarkably, it is found that the subspace signal mismatch affects the detection performance through a quantity, which can serve as a measure of the “distance” between the actual signal and nominal signal subspace. This generalizes the results for rank-one signals, e.g., [5,17–19].

The rest of the paper is organized as follows. Section 2 presents the signal model and related detectors. Section 3 derives the statistical distributions of the detectors and obtain the analytical PDs and PFAs. Numerical examples are given in Section 4. Finally, Section 5 concludes the paper.

Notations: Scalars are denoted by lightfaced lowercase letters, vectors by boldfaced lowercase letters, and matrices by boldfaced uppercase letters. $C_n^m = n!/[m!(n-m)!]$ is the binominal coefficient, $(h)_n = h(h+1)\cdots(h+n-1)$ is the Pochhammer symbol, $B(m,n) = (m-1)!(n-1)!/(m+n-1)!$ is the Beta function, $IG_{k+1}(a) = e^{-a}\sum_{m=0}^{k} a^m/m!$ is the incomplete Gamma function, and ${}_1F_1(a,b;x) = \sum_{n=0}^{\infty} [(a)_n x^n]/[(b)_n n!]$ is the confluent hypergeometric function of the first kind. $(\cdot)^{\mathrm{T}}$ stands for transpose and $(\cdot)^{\mathrm{H}}$ for conjugate transpose. $Pr(\cdot)$ is the probability of the event in the brackets. The notation $\sim$ denotes “be distributed as”, $\mathrm{CN}_N(\boldsymbol{\mu}, \boldsymbol{Q})$ denotes an $N\times 1$ complex circular Gaussian distribution with a mean $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{Q}$, $\mathrm{CW}_N(L, \boldsymbol{Q})$ denotes an $N\times N$ complex central Wishart distribution with L degrees of freedom (DOFs) and a scale matrix $\boldsymbol{Q}$. $\mathrm{CF}_{M,N}(\xi)$ denotes a complex noncentral F-distribution with M and N DOFs and a noncentrality parameter ξ. $\mathrm{CB}_{M,N}(\delta^2)$ denotes a complex noncentral Beta-distribution with M and N DOFs and a noncentrality parameter δ^2. When $\delta^2 = \xi = 0$ the two noncentral distributions above become central, written as $\mathrm{CF}_{M,N}$ and $\mathrm{CB}_{M,N}$, respectively. For two random variables α_1 and α_2, $E\{h(\alpha_1,\alpha_2)|\alpha_2\}$ is the conditional expectation of $h(\alpha_1,\alpha_2)$ with α_2 fixed, where $h(\alpha_1,\alpha_2)$ is a function of α_1 and α_2. $<\boldsymbol{C}>$ denotes the subspace spanned by the columns of $\boldsymbol{C}$, $\boldsymbol{P}_{\boldsymbol{C}} = \boldsymbol{C}(\boldsymbol{C}^H\boldsymbol{C})^{-1}\boldsymbol{C}^H$ is the orthogonal projection matrix onto $<\boldsymbol{C}>$, and $\boldsymbol{P}_{\boldsymbol{C}}^{\perp} = \boldsymbol{I}_N - \boldsymbol{P}_{\boldsymbol{C}}$. When $\boldsymbol{C}$ is positive definite, $\boldsymbol{C}^{1/2}$ denotes its square-root matrix, that is, if $\boldsymbol{D} = \boldsymbol{C}^{1/2}$ then $\boldsymbol{D}\boldsymbol{D} = \boldsymbol{C}$. Finally, $\boldsymbol{0}_{M\times N}$ is an $M\times N$ zero matrix, and $\boldsymbol{I}_N$ is an $N\times N$ identity matrix.

2. Signal model and related detectors

For a binary hypothesis test, an $N\times 1$ test data $\boldsymbol{x}$ in the signal-presence hypothesis H_1 can be expressed by

$$\boldsymbol{x} = \boldsymbol{s} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{n}$ is the noise, distributed as $\boldsymbol{n} \sim \mathrm{CN}_N(\boldsymbol{0}_{N\times 1}, \boldsymbol{R}_T)$, with $\boldsymbol{R}_T$ being an unknown positive definite matrix. The signal $\boldsymbol{s}$ is supposed to lie in the subspace $<\boldsymbol{H}>$, where $\boldsymbol{H}$ is an $N\times s$ full-column-rank matrix. Hence it can be represented as $\boldsymbol{s} = \boldsymbol{H}\boldsymbol{\theta}$, where the $s\times 1$ vector $\boldsymbol{\theta}$ stands for the unknown coordinate. In contrast, in the signal-absence hypothesis H_0, we have $\boldsymbol{x} = \boldsymbol{n}$. To estimate the unknown covariance matrix $\boldsymbol{R}$, it is often assumed that a set of training data $\boldsymbol{x}_l$, $l = 1,2,\ldots,L$, is available, and each of them is signal-free and only contains noise $\boldsymbol{n}_l$. Moreover, $\boldsymbol{n}_l$'s are mutually independent, distributed as $\boldsymbol{n}_l \sim \mathrm{CN}_N(\boldsymbol{0}_{N\times 1}, \boldsymbol{R})$. Hence, the detection problem can be formulated as

$$\begin{cases} H_0: \boldsymbol{x} = \boldsymbol{n}, & \boldsymbol{x}_l = \boldsymbol{n}_l, \quad l = 1,\ldots,L, \\ H_1: \boldsymbol{x} = \boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{n}, & \boldsymbol{x}_l = \boldsymbol{n}_l, \quad l = 1,\ldots,L. \end{cases} \tag{2}$$

In the homogeneous environment (HE) $\boldsymbol{R}_T = \boldsymbol{R}$, and the corresponding GLRT for the detection problem in (2), denoted as the SGLRT, is given by [7,8]

$$t_{SGLRT} = \frac{\tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}\tilde{\boldsymbol{x}}}{1 + \tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}^{\perp}\tilde{\boldsymbol{x}}}, \tag{3}$$

where $\tilde{\boldsymbol{x}} = \boldsymbol{S}^{-1/2}\boldsymbol{x}$, $\tilde{\boldsymbol{H}} = \boldsymbol{S}^{-1/2}\boldsymbol{H}$, and $\boldsymbol{S} = \boldsymbol{X}_L\boldsymbol{X}_L^{\mathrm{H}}$ is the sample covariance matrix, with $\boldsymbol{X}_L = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_L]$. The corresponding two-step GLRT, called the SAMF, in the HE is [10]

$$t_{SAMF} = \tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}\tilde{\boldsymbol{x}}. \tag{4}$$

In contrast, in the partially homogeneous environment (PHE), $\boldsymbol{R}_T = \sigma^2\boldsymbol{R}$ [20]. The scaling factor σ^2 is unknown, standing for the unknown power mismatch between the test and training data. For the detection problem in (2) in the PHE, the GLRT, referred to as the ASD, has the form [11]

$$t_{ASD} = \frac{\tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}\tilde{\boldsymbol{x}}}{\tilde{\boldsymbol{x}}^{\mathrm{H}}\tilde{\boldsymbol{x}}}, \tag{5}$$

which is statistically equivalent to

$$t_{ASD}' = \frac{\tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}\tilde{\boldsymbol{x}}}{\tilde{\boldsymbol{x}}^{\mathrm{H}}\boldsymbol{P}_{\tilde{\boldsymbol{H}}}^{\perp}\tilde{\boldsymbol{x}}}, \tag{6}$$

since $t_{ASD}' = (t_{ASD}^{-1} - 1)^{-1}$ can be taken as a monotonically increasing function of t_{ASD}.

Remarkably, the statistical performance of the SGLRT is exploited in [7,8], the SAMF in [9,10], and the ASD in [15,16], yet all under the assumption of no signal mismatch. In next section we will investigate the statistical performance of these three detectors with subspace signal mismatch in the HE.

3. Statistical performance analysis

When subspace signal mismatch occurs, the actual signal, denoted by $\boldsymbol{s}_0$, does not belong to the presumed signal subspace $<\boldsymbol{H}>$. To quantify the degree of mismatch, we introduce a generalized cosine squared (GCS), defined as

$$\cos^2\phi = \frac{\boldsymbol{s}_0^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{s}_0}{\boldsymbol{s}_0^{\mathrm{H}}\boldsymbol{R}^{-1}\boldsymbol{s}_0}, \tag{7}$$

where ϕ denotes the angle between $\boldsymbol{s}_0$ and $<\boldsymbol{H}>$ in the whitened space. It will be seen that the GCS in (7) plays a