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Direction finding and mutual coupling estimation for uniform rectangular arrays

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ABSTRACT

A novel two-dimensional (2-D) direct-of-arrival (DOA) and mutual coupling coefficients estimation algorithm for uniform rectangular arrays (URAs) is proposed. A general mutual coupling model is first built based on banded symmetric Toeplitz matrices, and then it is proved that the steering vector of a URA in the presence of mutual coupling has a similar form to that of a uniform linear array (ULA). The 2-D DOA estimation problem can be solved using the rank-reduction method. With the obtained DOA information, we can further estimate the mutual coupling coefficients. A better performance is achieved by our proposed algorithm than those auxiliary sensor-based ones, as verified by simulation results.

1. Introduction

Direction of arrival (DOA) estimation for two-dimensional (2-D) arrays is an important area of array signal processing and has received much attention in past years [1]. The well-known multiple signal classification (MUSIC) algorithm can be applied directly for 2-D estimation [2], but its computational complexity is very high due to the required 2-D spectral search. On the other hand, the UCA-ESPRIT and 2-D Unitary ESPRIT algorithms can pair the azimuth and elevation angles belonging to the same source automatically without 2-D spectral searching or iterative optimization procedures [3,4]. In [5], a polynomial root-finding-based method was proposed using two parallel ULAs, by decoupling the 2-D problem into two 1-D problems to reduce the computational complexity. Another computationally efficient method was proposed in [6], where the propagator method in [7] was employed based on two parallel ULAs. However, this method requires pair matching between the 2-D azimuth and elevation estimation results and may not work effectively for some situations. To overcome the problem in [6], an L-shaped array was employed instead in [8]. Based on such an L-shaped geometry, a 1-D searching algorithm without the need of pair matching was proposed in [9]. while the subspace-based algorithm in [10] requires neither constructing the correlation matrix of the received data nor performing singular value decomposition (SVD) of the correlation matrix and utilizes the conjugate symmetry property to enlarge the effective array aperture. Another computationally efficient algorithm for URA was proposed in [11], where the complex-valued covariance matrix and the complex-valued search vector are transformed into real-valued ones, and the 2-D problem is decoupled into two 1-D problems with real-valued computations.

However, for the above algorithms and methods to work, it is normally assumed that the exact array manifold is known in advance, which may not be practical in many applications due to the effect of mutual coupling. Similar to the 1-D case, the effect of unknown mutual coupling can cause severe performance degradation in 2-D DOA estimation [12,13]. As a result, some 2-D array calibration algorithms have been proposed. In [14], azimuth estimation is decoupled from elevation estimation and can be performed

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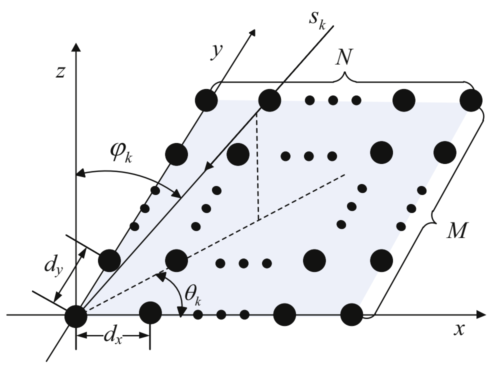

Fig. 1. Geometry of a URA with $M \times N$ sensors.

without the knowledge of mutual coupling, while for elevation estimation, a 1-D parameter search is performed and the elevation-dependent mutual coupling effect can be compensated effectively. In [15], a rank-reduction (RARE) algorithm for UCA was proposed based on the special structure of the coupling matrix considered in [16] and the result derived in [17]. In [18], two mutual coupling calibration methods were provided for uniform hexagon arrays (UHAs), one of which is also based on the method in [16], while the other is implemented by setting some auxiliary sensors. In [19], the mutual coupling model was extended to L-shaped arrays, where the mutual coupling effect is compensated using the outputs of properly chosen sensors and a rank-reduction propagator method is developed for joint estimation of both azimuth and elevation angles to avoid parameter pairing and 2D spectral search. To mitigate the effect of mutual coupling, the algorithm in [20] set the sensors on the array boundary to be auxiliary ones. The subarray's output and size are used to calculate the noise subspace and steering vector. The procedure of this algorithm is similar to the 2-D MUSIC algorithm. It obtains the DOAs through 2-D spectral searching by exploiting the orthogonality between the noise subspace and steering vector.

The auxiliary sensor-based algorithms, although effective in the presence of mutual coupling, share the common drawback that the effective aperture of the array is reduced. When considering mutual coupling between sensors farther apart, a larger number of auxiliary sensors are needed, which in turn reduces the number of sensors available for DOA estimation, since the total number of sensors is fixed. Therefore, the performance of these algorithms will deteriorate significantly when the size of original array is small or the mutual coupling effect is strong.

In this paper, we construct a general mutual coupling model for URAs using banded symmetric Toeplitz matrices and based on this model, we prove that the steering vector of such a URA in the presence of mutual coupling has a similar form to that of ULA using the method proposed in [21]; then, the rank-reduction method is introduced to estimate the azimuth and elevation angles, which are then used to obtain the unknown mutual coupling coefficients. As shown in our simulation results, the proposed algorithm can achieve a better performance than auxiliary sensor-based ones since it employs the full array aperture for DOA estimation.

The rest of this paper is organized as follows. In Section 2, the signal model in the presence of mutual coupling is introduced. The proposed DOA and mutual coupling coefficients estimation algorithm is presented with detailed analysis of the steering vector in Section 3. Simulation results are given in Section 4 and conclusions are drawn in Section 5.

Notations: $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{+}$ represent transpose, conjugate transpose and pseudo-inverse of a matrix or vector, respectively. $[\cdot]_{p,q}$ denotes the element at pth row and qth column of a matrix, and $\otimes$ denotes the Kronecker product.

2. Problem formulation with banded symmetric Toeplitz mutual coupling matrix

Consider K far-field narrowband signals $s_k(t)$, $k = 1, 2, \ldots, K$, with identical wavelength λ impinge on a URA of $M \times N$ omnidirectional sensors spaced by d_x in the x-axis direction and d_y in the y-axis direction, as shown in Fig. 1. The direction of arrival of the kth signal is denoted by (θ_k, φ_k), where θ_k and φ_k are the azimuth and elevation angles, respectively. The received data vector $\mathbf{x}(t)$ of the array at sample t can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_N(t), x_{N+1}(t), \ldots, x_{2N}(t), \ldots, x_{MN}(t)^{\mathrm{T}}]$ holding the MN received array signals, $\mathbf{A} = [\mathbf{a}(\theta_1, \varphi_1), \mathbf{a}(\theta_2, \varphi_2), \ldots, \mathbf{a}(\theta_K, \varphi_K)]^{\mathrm{T}}$ is the array manifold matrix, $\mathbf{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^{\mathrm{T}}$ is the source signal vector and $\mathbf{n}(t) = [n_1(t), \ldots, n_N(t), n_{N+1}(t), \ldots, n_{2N}(t), \ldots, n_{MN}(t)]^{\mathrm{T}}$ is the additive white Gaussian noise vector. The steering vector $\mathbf{a}(\theta_k, \varphi_k)$ can be modeled as

$$\mathbf{a}(\theta_k, \varphi_k) = \mathbf{a}_y(\theta_k, \varphi_k) \otimes \mathbf{a}_x(\theta_k, \varphi_k) \tag{2}$$

where

$$\mathbf{a}_y(\theta_k, \varphi_k) = [1, \beta_y(\theta_k, \varphi_k), \cdots \beta_y^{M-1}(\theta_k, \varphi_k)]^{\mathrm{T}} \tag{3}$$

$$\mathbf{a}_x(\theta_k, \varphi_k) = [1, \beta_x(\theta_k, \varphi_k), \cdots \beta_x^{N-1}(\theta_k, \varphi_k)]^{\mathrm{T}} \tag{4}$$

with

$$\beta_y(\theta_k, \varphi_k) = \exp\{j2\pi\lambda^{-1} d_y \ \sin(\theta_k) \sin(\varphi_k)\} \tag{5}$$

$$\beta_x(\theta_k, \varphi_k) = \exp\{j2\pi\lambda^{-1} d_x \ \cos(\theta_k) \sin(\varphi_k)\} \tag{6}$$

For simplified notation, the pair of angles (θ, φ) is omitted in the following when not causing any confusion.

Considering the effect of mutual coupling, (1) should be modified as

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{7}$$

where $\mathbf{C}$ denotes the mutual coupling matrix (MCM). As indicated in [16,20,22], the coupling between neighboring sensors with the same inter-element spacing is almost the same, while the magnitude of mutual coupling coefficients between two far apart elements would be so small that this effect can be ignored. Therefore, the mutual coupling of ULA can be modeled as a banded symmetric Toeplitz matrix. In [20], this model was extended to URAs assuming that each sensor is only affected by the 8 immediately