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Frequency estimation by two- or three-point interpolated Fourier algorithms based on cosine windows

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ABSTRACT

This paper investigates the frequency estimation of a complex sinusoid obtained by interpolating two or three samples of the Discrete-Time Fourier Transform (DTFT) of a signal weighted by a suitable cosine window. Versions of the algorithms based on both DTFT complex values and modules are considered and the expressions for the related frequency estimators are provided. Iterative procedures are used in order to minimize the estimator variance due to additive wideband noise. Furthermore, the accuracies of the proposed estimators are verified and compared each other and with state-of-the art algorithms by means of computer simulations when applied to noisy or noisy and harmonically distorted complex sinusoids, respectively.

1. Introduction

Real-time frequency estimation of a sinusoid is often required in many engineering applications such as communications, audio systems, radar, sonar, power systems, measurement and instrumentation. It is well-know that the peak location of the Discrete Time Fourier Transform (DTFT) of the overall signal represents the maximum likelihood frequency estimate of a sinusoid embedded in additive wideband Gaussian noise [1]. Such a peak can be effectively and accurately located using a two-step search procedure. In the first step (called coarse-search) the frequency bin corresponding to the Discrete Fourier Transform (DFT) sample of largest magnitude is determined through a simple maximum search routine. In the second step (called fine-search) an accurate estimate of the interbin frequency location is obtained. The sum of the two procedure outputs then provides the desired frequency estimate. Various methods have been proposed in the scientific literature for fine-search implementation of either complex sinusoids [2–11] or real sinusoids [1,12–20] by interpolating two or more DFT complex samples or modules. In particular, more than two DFT samples are used when estimating the frequency of a real sinusoid in order to reduce the effect of the spectral interference from the image component [17–20], but at the cost of an increased wideband noise sensitivity of the related frequency estimator [19]. Also, more DFT samples are used in [11] to reduce the gap between the performance of the estimator proposed in [6] and the related unbiased Cramer–Rao Lower Bound (CRLB).

Two- or three-point interpolated Fourier algorithms are often used when estimating the frequency of a complex sinusoid [2–10]. The effect of wideband noise on the estimates returned by two interpolation points algorithms is minimized when using the Aboutanios and Mulgrew (AM) algorithm [8]. That algorithm requires two DTFT

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samples located exactly halfway between adjacent DFT samples and it returns a frequency estimator with a variance very close to the related unbiased CRLB by using an iterative procedure. However, only two iterations suffice to reach asymptotic accuracy [8]. Two different but equally accurate versions of the AM algorithm are proposed, based on DTFT complex values or modules, respectively. Conversely, a bias-corrected three-point interpolated Fourier algorithm relying only on complex DFT samples has been proposed by Candan in [5], specifically designed for signal records with a small number of samples. It approaches the Jacobsen estimator [4] when the number of analyzed samples is quite high, as occurs in many engineering applications. Both the AM and the Jacobsen algorithms are based on the rectangular window since only complex sinusoids are considered and the effect on the returned estimator of the interference due to spectral leakage from other possible signal tones is assumed to be negligible with respect to the effect of wideband noise. However, signals affected by disturbance tones are often encountered in practical applications so techniques to reduce their detrimental effect on frequency estimates need to be applied. Very recently, a bias-corrected iterative three-point interpolated Fourier procedure based on windowed data has been proposed in [10]. It returns accurate estimates even when only few data samples are analyzed, while requiring a smaller processing effort than the procedure proposed in [16].

The aim of this paper is to generalize both AM and Jacobsen algorithms to the case when the acquired signal is multiplied by a generic cosine window [21], in order to reduce the effect of spectral leakage from interfering tones on the estimated frequency. In addition, the module based version of the three-point interpolated Fourier algorithm is introduced and the iterative procedure proposed in [8,10,11] to minimize the estimator variance due to wideband noise is applied to both versions of the three-point interpolated Fourier algorithm.

Two different versions of two- or three-point weighted interpolated Fourier algorithms minimizing the estimator variances (simply called MV-IpDTFT(2) and MV-IpDTFT(3), respectively) are proposed. It is worth noticing that the AM algorithm is the particular case of the MV-IpDTFT(2) algorithm based on the rectangular window, while the Jacobsen algorithm corresponds to the MV-IpDTFT(3) version 1 based on the rectangular window. The expressions of the proposed estimators and the related variances due to wideband noise are derived. Furthermore, the accuracies of the derived estimators are compared each other through computer simulations in the case of noisy or noisy and harmonically distorted complex sinusoids, respectively.

The remaining of the paper is organized as follows. In Section 2 the expressions of the estimators provided by the MV-IpDTFT(2) and MV-IpDTFT(3) algorithms related to the first iteration are derived. In addition, the steps required by the procedures related to the proposed algorithms are described. The expressions for the variances of the proposed estimators and the constraints under which windowing is advantageous are then derived in Section 3. The accuracies of the derived expressions for the estimator variances are verified through computer simulations in Section 4. Moreover, in this section the accuracies of the considered estimators are then analyzed and compared each other by means of computer simulations in the case of noisy or noisy and harmonically distorted complex sinusoids. Finally, Section 5 concludes the paper.

2. The proposed two- or three-point Fourier estimators

The analyzed signal is modeled as

$$x(m) = Ae^{j(2\pi f m+\varphi)} + e(m), \quad m = 0, 1, 2, \ldots, M-1 \tag{1}$$

where A, f, and φ are respectively the amplitude, the frequency, and the phase of the complex sinusoid, $e(\cdot)$ is a complex additive white Gaussian noise of zero mean and variance σ^2, and M is the number of acquired samples. The frequency f can be expressed as

$$f = \frac{f_{in}}{f_s} = \frac{\nu}{M} = \frac{l+\delta}{M}, \tag{2}$$

where f_{in} is the frequency of the continuous-time complex sinusoid, f_s is the sampling frequency, ν represents the number of acquired sinusoid cycles, l is its rounded value and δ $(-0.5 \le \delta < 0.5)$ is the differences between ν and l. Coherent sampling implies $\delta = 0$, but usually non-coherent sampling occurs in practice [22].

The analyzed signal (1) is weighted by a suitable window function $w(\cdot)$, so obtaining the signal: $x_w(m) = x(m)\cdot w(m)$, $m = 0, 1, \ldots, M-1$. Windows belonging to the cosine-class are usually employed [13,20]

$$w(m) = \sum_{h=0}^{H-1} a_h \cos\left(2\pi \frac{h}{M} m\right), \quad m = 0, 1, \ldots, M-1 \tag{3}$$

where $H \ge 1$ represents the number of window coefficients a_h, $h = 0, \ldots, H-1$. This class of windows will be used in the remaining of the paper.

The DTFT of the windowed signal $x_w(\cdot)$ is given by

$$X_w(\lambda) = AW(\lambda-\nu)e^{j\varphi} + E_w, \tag{4}$$

where $W(\cdot)$ is the DTFT of the adopted window, and $E_w(\cdot)$ is the DTFT of weighted wideband noise.

We assume that the wideband noise power is negligible as compared with the complex sinusoid one. Hence, the DTFT of the windowed signal can be expressed as

$$X_w(\lambda) \cong AW(\lambda-\nu)e^{j\varphi}. \tag{5}$$

Assuming that the number of acquired samples is high enough ($M \gg 1$), after some algebraic calculations, the expression of the DTFT of the window $w(\cdot)$ can be written as

$$W(\lambda) = \frac{M \sin(\pi\lambda)}{\pi} \sum_{h=0}^{H-1} (-1)^h a_h \frac{\lambda}{\lambda^2 - h^2} e^{-j\pi\lambda} = \tilde{W}(\lambda) e^{-j\pi\lambda}, \tag{6}$$

where

$$\tilde{W}(\lambda) \triangleq \frac{M \sin(\pi\lambda)}{\pi} \sum_{h=0}^{H-1} (-1)^h a_h \frac{\lambda}{\lambda^2 - h^2}. \tag{7}$$