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Error bound for joint detection and estimation of multiple targets with random finite set state and observation

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ABSTRACT

By the use of the random finite set (RFS) and information inequality, this paper studies the error bound for joint detection and estimation (JDE) of multiple targets in the presence of clutters and missed detections. The JDE here refers to determining the number of the targets and estimating the states of the existing targets. The proposed bound is obtained based on the optimal sub-pattern assignment (OSPA) distance rather than the usual Euclidean distance. Maximum a posterior (MAP) detection criteria and unbiased estimation criteria are used in deriving the bound. Then, the special case of the bound is discussed when neither clutters nor missed detections exist. Example 1 shows the variation of the bound with the probability of detection and clutter density. Example 2 verifies the effectiveness of the bound by indicating the performance limitations of three classical multi-target JDE algorithms, which are multiple hypothesis tracking (MHT) filter, probability hypothesis density (PHD) filter, and cardinalized PHD filter.

1. Introduction

Joint detection and estimation (JDE) of multiple targets refers to determining the number of the targets and estimating the states of the existing targets from the noisy, missed and cluttered observations [1–3]. It is an important research and attracts great interest. Many approaches to this have been proposed in recent years [4–6].

Obviously, it is very necessary and urgent to find an error (lower) bound for assessing the achievable performance of the multi-target JDE algorithms. The performance evaluation methods based on the Cramér–Rao lower bound (CRLB) [7–11] only refer to the estimation error but not the detection error. Therefore, it is very hard to extend CRLB to the problem of multi-target JDE because of the uncertainties of the target number and association relationship between targets and measurements ([12, pp. 315–335]).

Within the random finite set (RFS) [13] framework, Rezaeian and Vo [14] derived the JDE error bounds for a single target in the presence of clutters and missed detections. Tong et al. [15] presented a recursive form of the bound in the case of only missed detections, and then extended the result to the multi-target case under the assumption of no clutters or missed detections. However, the multi-target error bound in [15] does not actually include the detection error (that is, the detection problem no longer exists here) since a number of the estimated targets, true targets and measurements are obviously equal in this case. To the best of our knowledge, there has been no study on the error bounds of multi-target JDE with clutters or missed detections until now.

By the use of the RFS and information inequality ([16, pp. 169–171, 186]), this paper proposes the multi-target error JDE bound when clutters and missed detections simultaneously exist. Compared with the multi-target

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error bound in [15], the proposed bound is much more complicated because of the detection problem generated by clutters and missed detections as well as the interrelation between detection and estimation. In order to obtain the conclusion of this paper, we firstly model the states of the multiple targets as a multi-Bernoulli RFS [17]. A general observation model where measurements are affected by independent Poisson distributed clutters and missed detections is used to describe the likelihood function. Since the JDE error is the average distance between the true set and estimated set of multi-target states, it is defined by the optimal sub-pattern assignment (OSPA) distance [18,19] rather than the usual Euclidean distance in this paper. Maximum a posterior (MAP) detection criteria and unbiased estimation criteria are used in deriving the bound. Then, the proposed bound is discussed in the special case where neither clutters nor missed detections exist. We show that the bound in this case coincides with the multi-target error bound of [15], specialized to one time instant. Finally, two numerical examples are presented in simulations. Example 1 shows the variation of the bound with the probability of detection and clutter density. Example 2 verifies the effectiveness of the bound by indicating the performance limitations of three classical multi-target JDE algorithms, which are multiple hypothesis tracking (MHT) filter [20], probability hypothesis density (PHD) filter [13], and cardinalized PHD filter [21,22].

In the current set up of this paper, our attention is restricted to the static multi-target JDE problem. In the future work, the recursive extension of the result to the filtering problem will be studied by consideration of multi-target state evolution.

The rest of the paper is organized as follows. Section 2 describes the problem for multi-target JDE based on RFS. In Section 3, we present the error bound for multi-target JDE and discuss a special case of the bound. Two numerical examples are presented in Section 4. The conclusions and future work are given in Section 5. The relevant mathematical proof is presented in Appendix A.

2. Problem statement for multi-target JDE with RFS state and observation

Let $X=\{\boldsymbol{x}_t\}_{t=1}^{|X|}$ denote the set of multi-target states, where $|\cdot|$ is the cardinality of a set and $\boldsymbol{x}_t\in\mathcal{X}$ is the state vector of the tth target in the single-target state space $\mathcal{X}$. Since the paper is restricted to the static multi-target JDE problem, $\boldsymbol{x}_t$ denotes the two-dimensional or three-dimensional position vector here. It can be written as $\boldsymbol{x}_t=[x_t,y_t]^T$ or $\boldsymbol{x}_t=[x_t,y_t,z_t]^T$ in general, where x_t, y_t and z_t denote the positions in X-axis, Y-axis and Z-axis, respectively. X is modeled as a multi-Bernoulli RFS, which is a union of N independent Bernoulli RFSs $X^{(t)}$

$$X=\cup_{t=1}^{N}X^{(t)},\tag{1}$$

where N denotes the maximum number of the targets, the density of $X^{(t)}$ is

$$f\left(X^{(t)}\right)=\begin{cases}1-r_t, & X^{(t)}=\varnothing,\\ r_t f_t(\boldsymbol{x}_t), & X^{(t)}=\{\boldsymbol{x}_t\},\end{cases}\tag{2}$$

where $r_t\in(0,1)$ denotes the probability of $X^{(t)}\neq\varnothing$, $f_t(\boldsymbol{x}_t)$ (defined on $\mathcal{X}$) denotes the density of $\boldsymbol{x}_t$. The density of multi-Bernoulli RFS X is completely described by the parameter set $\left\{\left(r_t,f_t(\cdot)\right)\right\}_{t=1}^{N}$ with the mean cardinality $\sum_{t=1}^{N}r_t$ [17]

$$f(X)=\pi(\varnothing)\sum_{1\le j_1\neq\cdots\neq j_{|X|}\le N}\prod_{t=1}^{|X|}\frac{r_{j_t}f_{j_t}(\boldsymbol{x}_t)}{1-r_{j_t}},\tag{3}$$

$$\pi(\varnothing)=\prod_{t=1}^{N}(1-r_t).\tag{4}$$

Let $Z=\{\boldsymbol{z}_b\}_{b=1}^{|Z|}$ denote the set of measurements received by a sensor, where $\boldsymbol{z}_b\in\mathcal{Z}$ is the bth measurement vector in the single-measurement space $\mathcal{Z}$. The measurement originates either from target or from random clutter. Moreover, the target-generated measurements are independent and indistinguishable from the clutters. Hence, Z is described as

$$Z=Z_\Theta(X)\cup Z_C,\tag{5}$$

where Z_C denotes the clutter set, $Z_\Theta(X)$ denotes the measurement set originated from the targets with the state set X, $Z_\Theta(X)$ is a union of $|X|$ independent RFS $Z_\Theta(\boldsymbol{x}_t)$

$$Z_\Theta(X)=\cup_{t=1}^{|X|}Z_\Theta(\boldsymbol{x}_t),\tag{6}$$

where $Z_\Theta(\boldsymbol{x}_t)$ denotes the measurement set originated from the target $\boldsymbol{x}_t$, $t=1,2,...,|X|$. The target $\boldsymbol{x}_t$ either generates a measurement $\boldsymbol{z}_b$ $(b=1,2,...,|Z|)$ with the detection probability $p_D(\boldsymbol{x}_t)\in(0,1]$, or is missed with the probability $1-p_D(\boldsymbol{x}_t)$. So $Z_\Theta(\boldsymbol{x}_t)$ can also be modeled as a Bernoulli RFS with the density

$$f(Z_\Theta(\boldsymbol{x}_t))=\begin{cases}1-p_D(\boldsymbol{x}_t), & Z_\Theta(\boldsymbol{x}_t)=\varnothing,\\ p_D(\boldsymbol{x}_t)g\left(\boldsymbol{z}_b|\boldsymbol{x}_t\right), & Z_\Theta(\boldsymbol{x}_t)=\{\boldsymbol{z}_b\},\end{cases}\tag{7}$$

where $g\left(\boldsymbol{z}_b|\boldsymbol{x}_t\right)$ denotes the single-target single-measurement likelihood.

The clutter set Z_C is modeled as a Poisson RFS with the intensity

$$\upsilon(\boldsymbol{z})=\lambda f_C(\boldsymbol{z}),\tag{8}$$

where λ is the average clutter number, $f_C(\cdot)$ is the density of clutter spatial distribution.

The multi-target JDE is to derive the estimated set $\hat{X}(Z)=\{\hat{\boldsymbol{x}}_t(Z)\}_{t=1}^{|\hat{X}(Z)|}$ of multi-target states based on the measurement set Z, which actually means the joint estimation of target number and states. We aim to find a performance limit for the multi-target JDE, which is measured by average error between X and $\hat{X}(Z)$.

3. Error bound for multi-target JDE

For defining the multi-target JDE error within the RFS framework, the definition of set integral ([23, pp. 141–144]) is firstly presented: for any real-valued function $\eta(Y)$ of a finite-