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On imperfect pricing in globally constrained noncooperative games for cognitive radio networks

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ABSTRACT

Pricing is often used in noncooperative games or Nash equilibrium problems (NEPs) to meet global constraints in cognitive radio networks. In this paper, we analyze the pricing mechanism for a class of solvable NEPs with global constraints, called monotone NEPs. In contrast to the ideal assumption of perfect measure of pricing functions, in practice pricing functions are often imperfectly known and subject to uncertainty. We theoretically analyze the impacts of bounded uncertainty and price-updating step sizes of imperfect pricing in globally constrained NEPs for cognitive radio networks.

1. Introduction

Noncooperative games are also called Nash equilibrium problems (NEPs) [1] that characterize conflicts among interacting decision-makers called players, where each player is regarded to be rational and wishes to selfishly optimize his own payoff. Such game theoretical models have been widely applied to communications and signal processing systems where conflicts or competition are inevitable, for example interference among wireless links (see a special issue [2] on game theory).

The solution to an NEP, i.e., Nash equilibrium (NE), is a point at which no player can gain or achieve a better payoff by unilaterally changing his strategy. In practice, such a solution may be obtained via the best-response algorithms [1], in which players simply optimize their own payoff given the strategies of the others according to a prescribed schedule, e.g. a sequential order. One example is the iterative waterfilling algorithm that arose in power control for digital subscriber lines [3]. However, due to players' selfish behaviors, the NE is often socially inefficient in the sense that global requirements are often unsatisfied.

A common way to improve the social efficiency of the NE is to use pricing that penalizes players' selfish behaviors through some pricing function [4]. As an important application, several pricing mechanisms [5,6] have been proposed for cognitive radio networks (CRNs) where secondary users (SUs) compete the resources of primary users (PUs) but have to satisfy some global interference constraints imposed by PUs. It was shown in [5,6] that the pricing mechanisms can be distributedly implemented and enforce the players (SUs) to meet the global constraints.

The NEP based methods rely on local information measurements, which are, however, often imperfect in CRNs. For example, the channel state information (CSI) between SUs and PUs [7], the interference plus noise (IPN)

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[8], or the best response [9] could be imperfectly measured by SUs. In particular, [10,11] considered imperfect SU-to-PU CSI in pricing NEP designs for CRNs. Although these works considered imperfect information measurements by SUs, they all assumed that pricing functions can be perfectly measured by PUs. In practice, however, pricing functions are more likely to be imperfectly measured since any imperfect local measurement (of, e.g., CSI) by SUs could lead to imperfect measurement of pricing functions by PUs. To the best of our knowledge, imperfect pricing in globally constrained NEPs for CRNs has not been addressed yet.[1]

In this paper, we would like to investigate the influence of imperfect pricing in a class of solvable NEPs called monotone NEPs [13] with achievable solutions by the best-response algorithms. The monotonicity leads to a favorable property called co-coercivity that facilitates the NEPs to meet global constraints with perfect pricing. Then, we consider a more practical situation where pricing functions are imperfectly measured and subject to bounded uncertainty. We theoretically analyze the global impacts of bounded uncertainty and choices of step sizes for the pricing updating mechanism in globally constrained NEPs. The studied framework is then demonstrated through numerical examples in CRNs.

2. Nash equilibrium problem with pricing for CRNs

Consider a CRN of K PUs and N SUs over L-subcarrier interference channels. Let h_{ji}^l be the channel between the secondary transmitter j and the secondary receiver i on subcarrier l, and g_{ik}^l be the channel between the transmitter of SU i and the receiver of PU k on subcarrier l. Let $\mathbf{p}_i = (p_i^l)_{l=1}^L$ with p_i^l being the power allocated by SU i on subcarrier l. Then, the information rate of SU i is given by

$$r_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{l=1}^{L} \log\left(1 + \frac{|h_{ii}^l|^2 p_i^l}{\sigma_i^l + \sum_{j \neq i} |h_{ji}^l|^2 p_j^l}\right), \tag{1}$$

where σ_i^l is the noise power on subcarrier l. Observe that $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ depends not only on SU i's transmit power $\mathbf{p}_i$ but also on the transmit power $\mathbf{p}_{-i} = (\mathbf{p}_j)_{j \neq i}$ of the other SUs. A popular way to design strategies of cocurrent transmission of all SUs is to exploit noncooperative game, also known as NEP.

An NEP consists of three components [1]: players, payoff (or cost) functions, and strategy sets. Here, the players are $i = 1, \ldots, N$ SUs and the payoff function of player (or SU) i is his information rate $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$. The strategy set of player i is given by $\mathcal{P}_i = \{\mathbf{p}_i : \sum_{l=1}^L p_i^l \le P_i\}$, which limits the transmit power of SU i below P_i. Then, in the NEP, each player i would aim to maximize his information rate $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ by choosing a proper power strategy from $\mathcal{P}_i$. The solution to the NEP, also known as Nash Equilibrium (NE), is a strategy profile $\mathbf{p} = (\mathbf{p}_i)_{i=1}^N$, at which no player can gain or achieve a larger rate by unilaterally changing his strategy.

[1] In the case that payoff functions are not fully known, learning mechanisms can be used [12].

The above-mentioned (non-priced) NEP is built on the selfish nature of the players and thus may lead to socially inefficient NE in the sense that, at the NE, either some global constraint is violated or overall system performance is not good. Specifically, to protect PUs' communications in the CRN, the SUs must satisfy the global interference constraints

$$\sum_{i=1}^{N} \sum_{l=1}^{L} |g_{ik}^l|^2 p_i^l \le I_k, \quad k = 1, \ldots, K \tag{2}$$

which restrict the interference caused by all SUs at each PU k below the given threshold I_k. Each SU selfishly optimizing his own payoff would lead to violations of the global interference constraints.

An effective way to tackle this issue is to introduce pricing into NEPs and properly penalize players' selfish behaviors. For each PU k, we can define the pricing function $z_k(\mathbf{p}) = \sum_{i=1}^N \sum_{l=1}^L |g_{ik}^l|^2 p_i^l - I_k$, and associate each pricing function $z_k(\mathbf{p})$ with a price $\lambda_k \ge 0$. Then, the priced NEP can be mathematically formulated as

$$(\mathcal{G}_\lambda): \underset{\mathbf{p}_i \in \mathcal{P}_i}{\text{maximize}}\ r_i(\mathbf{p}_i, \mathbf{p}_{-i}) - \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p}), \quad \forall i \tag{3}$$

where $\mathbf{z}(\mathbf{p}) = (z_k(\mathbf{p}))_{k=1}^K$ and $\boldsymbol{\lambda} = (\lambda_k)_{k=1}^K$. One can naturally interpret λ_k as the price of violating the interference constraint $z_k(\mathbf{p}) \le 0$. Let $g_i(\mathbf{p}_i, \mathbf{p}_{-i}) = r_i(\mathbf{p}_i, \mathbf{p}_{-i}) - \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p})$. Then, the NE of $\mathcal{G}_\lambda$ is given by a point $\mathbf{p}^*$ such that $g_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*) \ge g_i(\mathbf{p}_i, \mathbf{p}_{-i}^*)$, $\forall \mathbf{p}_i \in \mathcal{P}_i$ for $i = 1, \ldots, N$. By properly choosing $\boldsymbol{\lambda}$, the global interference constraints can be satisfied at the NE $\mathbf{p}^*$. Therefore, one shall expect

$$\boldsymbol{\lambda} \ge \mathbf{0}, \quad \mathbf{z}(\mathbf{p}^*) \le \mathbf{0}, \quad \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p}^*) = 0 \tag{4}$$

where the last condition simply says if the interference constraint is satisfied then no pricing is needed. We term $\mathcal{G}_\lambda$ and (4) a priced NEP and $(\boldsymbol{\lambda}^*, \mathbf{p}^*)$ the pricing equilibrium (PE) if the price vector $\boldsymbol{\lambda}^*$ satisfying (4) at the NE $\mathbf{p}^*$ of $\mathcal{G}_{\lambda^*}$. It was shown in [5,6] that the priced NEP approach leads to a nice distributed network design.

3. Best-response and pricing algorithms

Searching the PE of a priced NEP includes actually two parts: choose proper prices $\boldsymbol{\lambda}$ and find the NE of $\mathcal{G}_\lambda$ with given $\boldsymbol{\lambda}$, both depending on the properties of the strategy sets, the payoff functions, and the pricing functions. For the considered CRN we have the following properties: (1) $\mathcal{P}_i$ is a convex compact set; (2) $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ is twice differentiable and convex in $\mathbf{p}_i$ for $\forall i$; (3) $z_k(\mathbf{p})$ is convex in $\mathbf{p}$ for $\forall k$. We also introduce $\mathbf{F}(\mathbf{p}) = (-\nabla_{\mathbf{p}_i} r(\mathbf{p}))_{i=1}^N$ and $\mathcal{P} = \prod_{i=1}^N \mathcal{P}_i$, where $\nabla_{\mathbf{p}_i} r(\mathbf{p})$ is the gradient of $r(\mathbf{p})$ with respect to $\mathbf{p}_i$. With the above properties, given any $\boldsymbol{\lambda} \ge \mathbf{0}$ the NEP $\mathcal{G}_\lambda$ is guaranteed to possess at least one solution [5].

To solve the NEP $\mathcal{G}_\lambda$ with given $\boldsymbol{\lambda}$, we introduce an important concept called the strong monotonicity: $F(\mathbf{p})$ is strongly monotone on $\mathcal{P}$ if $(\mathbf{x}-\mathbf{y})^T(F(\mathbf{x}) - F(\mathbf{y})) \ge \alpha_s \|\mathbf{x}-\mathbf{y}\|^2$ for $\forall \mathbf{x}, \mathbf{y} \in \mathcal{P}$ with a positive constant α_s. To verify the strong monotonicity of $F(\mathbf{p})$, let us define an $N \times N$ matrix $\boldsymbol{\Phi}$ with $[\boldsymbol{\Phi}]_{ij} = -\sup_{\mathbf{p} \in \mathcal{P}} \| -\nabla^2_{\mathbf{p}_i \mathbf{p}_j} r(\mathbf{p}) \|_2$ for $i \neq j$ and $[\boldsymbol{\Phi}]_{ii} = \inf_{\mathbf{p} \in \mathcal{P}} \lambda_{\min}(-\nabla^2_{\mathbf{p}_i} r(\mathbf{p}))$, where $\|\cdot\|_2$ de notes the spectral norm and $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of