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An optimal variable step-size affine projection algorithm for the modified filtered-x active noise control

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ABSTRACT

This paper introduces an optimal variable step-size affine projection algorithm for the modified filtered-x active noise control systems. First, the recursion form of the error covariance from the tap weight update equation is constructed, not ignoring the dependency between the estimation error and the secondary noise signal. Such consideration has not been concerned previously for the analysis of the modified filtered-x affine projection algorithm. Second, a recursion form of the mean square deviation is derived from that of the error covariance. From the recursion form, an optimal step size is decided to get the fastest convergence rate. Both the recursion forms of the mean square deviation and the optimal step size require scalar additions and multiplications that do not contribute to the overall complexity seriously. The simulation results on the active noise control environments show both fast convergence rate and low steady-state error.

1. Introduction

The goal of active noise control (ANC) is to cancel out the unwanted disturbance signal using the acoustic signal. The acoustic signal is generated from controllable secondary source. With feedback or feed forward control using electrical signals, the ANC system generates acoustic signals for destructive interference at the sound fields [1]. The unwanted disturbance signals for destructive interference have long wavelengths with low frequency. The low frequency noise is hard to be suppressed by conventional passive ways. Thus, the method named as the ANC is developed to efficiently remove the acoustic noise.

In ANC systems, the least mean squares (LMS) algorithm has been used as the form of filtered-x LMS algorithm (FxLMS), for low cost and complexity. To improve the performance, the FxLMS algorithm is modified by various ways: the generalized FxLMS recursion [2], the convex algorithm [3], and variable step-size algorithm [4]. However, a drawback of the FxLMS algorithm is that its convergence rate gets worse in correlated input environments.

Typically, due to the secondary path, the ANC system shows the similar effect with correlated inputs. The effect is caused from the pre-filtered input source to the adaptive filter to update coefficient equation, although the input signal to the copy of the adaptive filter is not filtered to the estimate of the secondary path. Thus the FxLMS algorithm shows poor convergence rate at the ANC system. To overcome the weak point of the FxLMS algorithm for the ANC system, the affine projection algorithm (APA) has been used as the filtered-x affine projection (FxAP) algorithm [5] and modified filtered-x affine projection (MFxAP) algorithm [6].

The performance of FxAP algorithm is shown by analyzing the transient and the steady states [7,8]. Each literature analyzes the transient analysis of the conventional FxAP (CFxAP) algorithm, and the steady-state mean square performance of both the CFxAP algorithm and the MFxAP algorithm. Both analyses use the energy conservation relation, which has been already used to analyze the mean-square performa nce of a family of APA [9]. They show good agreement with

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simulations in the ANC system environments. However, those analyses ignore the dependency between the weight-error vector and past noise. Additionally, using the energy conservation relation is too complex to apply at ANC systems for real-time algorithms.

On the other hand, a different approach was presented to analyze the mean-square deviation (MSD) of APA, considering the dependency between the weight-error vector and past noise [10]. It constructs the propagation model of the error covariance having lower complexity and showing great simulation result on analysis in better agreement than other analyses. Based on the analysis, the optimal step size is derived to get the best performance [11]. However, there are just few literatures of improving the MFxAP algorithm to get fast convergence rate and low steady state error [12], compared to APA [13–15]. Thus, to obtain those requirements simultaneously, in this paper, an optimal step size of the MFxAP algorithm is suggested, applying the analysis of APA [10].

This paper uses the optimal step size algorithm of the APA [10]. The proposed algorithm is developed by analyzing the MSD of the basic coefficient update equation of the MFxAP algorithm, which is different from that of the APA. It considered the difference between the APA and the MFxAP algorithm, including the pre-filtered input from the estimate of the secondary path. Full theoretical justification will be provided for the superior performance of the proposed algorithm, with some different proofs from [10] for better way. The non-stationary condition, which is often ignored and is essential for real implementation of the ANC, is also considered. Furthermore, efficiency consideration is also applied to the proposed algorithm to reduce the computational complexity.

This paper is organized as follows. Section 2 describes the system of the FxAP algorithm in the ANC environments. Section 3 presents a recursion form of the error covariance considering the dependency between the weight-error vector and the secondary noise vector for the MFxAP algorithm. Based on the recursion form, Section 4 analyzes the MSD and constructs the optimal step size of the MFxAP algorithm. Non-stationary consideration is presented in Section 5, and finally, Section 6 shows some simulation results verifying the performance and properties of the proposed optimal step size algorithm.

2. The filtered-x APA

For the ANC systems, each structure of the CFxAP algorithm and the MFxAP algorithm is depicted in Figs. 1 and 2, respectively. First, the primary noise signal, which is unwanted signal, passes through the unknown primary path. After that, the adaptive filter output $\mathbf{y}_i$ is added to the passed signal and it goes through the secondary path $\mathbf{h}$. The secondary path is considered as a finite impulse response (FIR) filter of different lengths from the primary path. The primary path is determined by assuming that the algorithm converges to the optimal solution $\mathbf{w}_o \in \mathcal{R}^{n\times 1}$. Thus, the desired signal $\mathbf{d}_i$ which has to be canceled out is given by $\mathbf{d}_i = -\mathbf{V}_i\mathbf{w}_o$. In practice, however the secondary noise signal should be considered, and the disturbance signal is defined with the projection order $M \geq 1$ as

$$\mathbf{d}_i = -\mathbf{V}_i^T\mathbf{w}_o + \mathbf{r}_i, \tag{1}$$

where

$$\mathbf{V}_i \triangleq [\mathbf{v}_i \ \ \mathbf{v}_{i-1} \ \ \dots \ \ \mathbf{v}_{i-M+1}] \in \mathcal{R}^{n\times M}, \tag{2}$$

$$\mathbf{v}_i \triangleq [v_i \ \ v_{i-1} \ \ \dots \ \ v_{i-n+1}]^T \in \mathcal{R}^{n\times 1}, \tag{3}$$

and v_i is a signal passed through the estimate of secondary path $\hat{h} \in \mathcal{R}^{L\times 1}$ as $v_i = x_i * \hat{h}$. For the estimation of secondary path, it is assumed to be done perfectly in this paper. The secondary noise signal $\mathbf{r}_i \in \mathcal{R}^{M\times 1}$ is a zero mean white Gaussian noise vector with variance σ_r^2.

The basic coefficient update equation of FxAP can be written with the following recursion formula:

$$\hat{\mathbf{w}}_{i+1} = \hat{\mathbf{w}}_i - \mu_i \mathbf{V}_i \left(\mathbf{V}_i^T\mathbf{V}_i\right)^{-1}\mathbf{e}_i, \tag{4}$$

where $\hat{\mathbf{w}}_i \in \mathcal{R}^{n\times 1}$ is the impulse response of the adaptive filter, and $\mu_i \in (0,1]$ is a variable step size. For the use of $\mathbf{e}_i$, the CFxAP algorithm uses samples of the error signal e_i, at the error microphone, because it cannot use the disturbance signal d_i. However, the MFxAP algorithm can reconstruct the disturbance signal and the error signal by using the output of the estimate of secondary path. Thus, in the MFxAP algorithm, the error vector $\mathbf{e}_i$ is given by

$$\mathbf{e}_i = \mathbf{d}_i + \mathbf{V}_i^T\hat{\mathbf{w}}_i. \tag{5}$$

The plus sign after $\mathbf{d}_i$ means the addition at a sensor like a microphone, and the summation is not an electrical sum but an acoustic sum, so Eq. (5) can be used in the MFxAP algorithm, but not in the CFxAP algorithm. On the other hand, the MFxAP algorithm recovers the desired signal from the error signal using the output from the secondary path estimator after the adaptive filter, so it can use Eq. (5). In this paper, only MFxAP algorithm is handled, because it shows better convergence rate than CFxAP algorithm.

3. Augmented modeling of the modified filtered-x APA

From the tap weight update equation, the recursion form of the tap error vector $\tilde{\mathbf{w}}_i = \mathbf{w}_o - \hat{\mathbf{w}}_i$ is formulated as

$$\tilde{\mathbf{w}}_{i+1} = \tilde{\mathbf{w}}_i + \mu_i \mathbf{V}_i \left(\mathbf{V}_i^T\mathbf{V}_i\right)^{-1}\left(-\mathbf{V}_i^T\tilde{\mathbf{w}}_i + \mathbf{r}_i\right), \tag{6}$$

and let the transition matrix be

$$\mathbf{\Phi}(i+1,i) = \mathbf{I}_n - \mu_i \mathbf{V}_i\left(\mathbf{V}_i^T\mathbf{V}_i\right)^{-1}\mathbf{V}_i^T, \tag{7}$$

where $\mathbf{I}_a$ is an identity matrix in $\mathcal{R}^{a\times a}$. Then, the transition matrix has two properties such as

$$\mathbf{\Phi}(i,i) = \mathbf{I}_n, \quad \mathbf{\Phi}(i,j) = \mathbf{\Phi}(i,k)\mathbf{\Phi}(k,j) \tag{8}$$

With the transition matrix, the recursion form of tap error vector can be rewritten as

$$\begin{bmatrix} \mathbf{r}_{i+1} \\ \tilde{\mathbf{w}}_{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{Z} & \mathbf{0} \\ \mu_i \mathbf{V}_i\left(\mathbf{V}_i^T\mathbf{V}_i\right)^{-1} & \mathbf{\Phi}(i+1,i) \end{bmatrix} \begin{bmatrix} \mathbf{r}_i \\ \tilde{\mathbf{w}}_i \end{bmatrix} + \begin{bmatrix} r_{i+1} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

where $\mathbf{Z} = \begin{bmatrix} \mathbf{0} & 0 \\ \mathbf{I}_{M-1} & \mathbf{0} \end{bmatrix} \in \mathcal{R}^{M\times M}$, and $\mathbf{0}$ is the zero matrix of appropriate dimensions.

The MSD matrix at each iteration i is defined as $E\left[\tilde{\mathbf{w}}_i^T\tilde{\mathbf{w}}_i\right] = \mathrm{Tr}(\mathbf{P}_i)$, where $E(\cdot)$ is the notation of expectation. $\mathbf{P}_i$ is the covariance matrix of tap error vector which can be