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General similar sensing matrix pursuit: An efficient and rigorous reconstruction algorithm to cope with deterministic sensing matrix with high coherence

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ABSTRACT

In this paper, a novel algorithm, called the general similar sensing matrix pursuit (GSSMP), is proposed to deal with the deterministic sensing matrix with high coherence. First, the columns of the sensing matrix are divided into a number of similar column groups based on the similarity distance. Each similar column group presents a set of coherent columns or a single incoherent column, which provides a unified frame work to construct the similar sensing matrix. The similar sensing matrix is with low coherence provided that the minimum similar distance between any two condensed columns is large. It is proved that under appropriate conditions the GSSMP algorithm can identify the correct subspace quite well, and reconstruct the original K-sparse signal perfectly. Moreover, we have enhanced the proposed GSSMP algorithm to cope with the unknown sparsity level problem, by testing each individual contributing similar column group one by one to find the true vectors spanning the correct subspace. The simulation results show that the modified GSSMP algorithm with the contributing similar column group test process can estimate the sparse vector representing the radar scene with an unknown number of targets successfully.

1. Introduction

Compressed sensing has received considerable attention recently, and has been studied in diverse fields, e.g., image processing [1], underwater acoustic communication [2], wireless communication [3] and radar [4–6]. The central goal of compressed sensing is to capture attributes of a signal using very few measurements. In most work to date, this broader objective is exemplified by the important special case in which a K-sparse vector $\mathbf{x} \in \Re^N$ (with N large) is to be reconstructed from a small number M of linear measurements with $K < M < N$. K-sparse signals are the signals that can be represented by K significant coefficients over an N-dimensional basis. This can be compactly described by

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{e}, \tag{1}$$

where $\mathbf{y} \in \Re^M$ denotes a measurement vector, $\mathbf{\Phi}$ represents an $M \times N$ sensing matrix, and $\mathbf{e} \in \Re^M$ is an arbitrary noise vector with $\|\mathbf{e}\|_2 \le \epsilon$, where ϵ is the bound on the noise magnitude.

In compressed sensing, one of the well-studied conditions on the sensing matrix which guarantees stable recovery for a number of reconstruction algorithms is the

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restricted isometry property (RIP) [7,8]. However, in practice, it is very hard to check whether a sensing matrix satisfies RIP or not.

Coherence, the maximal correlation between two columns in a sensing matrix, is also a well-known performance measure for sensing matrices. For a matrix $\mathbf{\Phi}$ with columns $\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_N$, the coherence of $\mathbf{\Phi}$ is defined as

$$\mu(\mathbf{\Phi}) = \max_{1 \le i,j \le N \text{ and } i \ne j} \frac{|\boldsymbol{\varphi}_i^T \boldsymbol{\varphi}_j|}{\|\boldsymbol{\varphi}_i\|_2 \cdot \|\boldsymbol{\varphi}_j\|_2}. \quad (2)$$

Coherence plays a central role in the sensing matrix construction, because small coherence implies the RIP [9].

In the early work of compressed sensing, the entries of the sensing matrix are generated by an independent identically distributed (i.i.d.) Gaussian or Bernoulli process, or from random Fourier ensembles [10–12]. The role of random measurement provides the worst case performance that guarantees in the context of an adversarial signal/error model. Random sensing matrices are easy to construct, and are $2K$-RIP with high probability [12]. However, the random matrices are often not feasible for real-world applications due to the cost of multiplying arbitrary matrices with signal vectors of high dimension, and there is no guarantee that a specific realization works perfectly for the reconstruction.

With the application area of compressed sensing extended to wider fields, the random sensing matrix is being replaced by the deterministic sensing matrix. Recent research has focused on the construction of the deterministic matrix which often exhibits considerable structure [13]. In [14], the authors propose a new deterministic low-storage construction of compressive sampling matrices based on classical finite-geometry generalized polygons. This can be seen as a foundation on deterministic sensing matrices and reconstruction. The connection between sensing matrices and coding theory can be traced to [15], where the theory of finite classical generalized polygons is utilized to derive and study low-density parity-check (LDPC) codes. Among the approaches for deterministic matrix construction, the Vandermonde matrices seem to be good options, since any K columns of an $M \times N$ Vandermonde matrix are linearly independent. However, when N increases, the constant δ_K rapidly approaches 1 and some of the $M \times K$ submatrices become ill-conditioned [16]. In [17], the second order Reed–Muller codes are used to construct bipolar matrices. However, they lack a guarantee on the RIP order. In [18], the authors propose a series of deterministic sensing matrices, the binary, bipolar, and ternary compressed sensing matrices which satisfy the RIP condition.

The key concept of coherence is extended to pairs of orthonormal bases. This enables a new choice for the sensing matrices: one simply selects an orthonormal basis that is incoherent with the sparsity basis, and obtains measurements by selecting a subset of the coefficients of the signal in the chosen basis [19]. This approach has applications in radar systems [6,20], where an additional sensing matrix $\mathbf{H}$ is introduced and the received signal is compressed further by making nonadaptive, linear projections of the direct data sampled at the Nyquist frequency. However, neither of these algorithms mention the hardware implementation of the additional sensing matrix, which is very complex and expensive.

In this work, we focus on the deterministic sensing matrix built directly on the real acquisition systems, e.g., radar systems [21], and sensor array systems [22], which have been widely used in underwater acoustics and wireless communications. In the context of phased array radar system based on space time adaptive processing (STAP) technique [21], the sensing matrix is composed of spatial-Doppler steering vectors in columns, which is deterministic in nature. As the resolution of the angle-Doppler plane becomes finer, the coherence between the columns of the sensing matrix increases, thereby degrading the reconstruction reliability and performance. Similarly, for the sensor array system [22], the sensing matrix is composed of DOA steering vectors in columns. Increasing the resolution of the DOA angle plane leads to finer gridding, which increases the correlation between the basis elements of the sensing matrix.

Only a few papers have been published on the deterministic sensing matrix with high coherence [23–25]. Sparse Bayesian learning (SBL) algorithm [26–28] is capable of handling the sensing matrix with high coherence, and has been applied in the passive SAR radar system to improve the imaging resolution [23]. In [24], a novel approach based on the SBL algorithm is proposed for sparse nonstationary signal reconstruction using multiple windows.

In our previous work [25], a novel algorithm, called the similar sensing matrix pursuit (SSMP), is proposed to cope with the deterministic sensing matrix with high coherence. The proposed algorithm builds a similar sensing matrix based on the original sensing matrix, which has low coherence. A subspace pursuit (SP) algorithm is then used to find a rough estimate of the true support set, which contains the indices of the columns that contribute to the original sparse vector. Three kinds of structures of the estimated support set are considered, and three individual refined estimation processes are carried out under these three conditions. The proposed algorithm obtains much better performance while coping with a deterministic sensing matrix with high coherence, compared with the SP and basis pursuit (BP) algorithms. However, the proposed algorithm is heuristic in nature, which implies that the original K-sparse vector can be reconstructed based on the similar sensing matrix with high probability, and there is no rigorous proof for this conclusion. Moreover, two thresholds have to be set in the algorithm. One threshold is used to distinguish the incoherent columns from the coherent columns, and the other is used to divide the coherent columns into separate similar column groups. The setting of the two thresholds is very tight, which limits the application area of the SSMP algorithm. This method is complex and impractical for real-wold problems, since the incoherent and coherent columns are treated separately and three individual refined estimation processes under three different conditions have to be considered.

This paper presents a rigorous version of the SSMP algorithm in a unified framework, which is named as general