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Embedded cubature Kalman filter with adaptive setting of free parameter

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A B S T R A C T

The choice of free parameter in embedded cubature Kalman filter (ECKF) is important, and it is difficult to choose an optimal value in practice. To solve this problem, an adaptive method is proposed to determine the value of free parameter of ECKF based on maximum likelihood criterion. By incorporating this method in the third-degree ECKF, a new third-degree adaptive ECKF (AECKF) algorithm is obtained. To further improve the accuracy of the third-degree AECKF, a new fifth-degree AECKF based on the fifth-degree embedded cubature rule is developed. Simulation results show that the proposed algorithms have higher estimation accuracy than existing methods.

1. Introduction

Embedded cubature Kalman filter (ECKF) using the third-degree embedded cubature rule (ECR) has been gaining more attention because it has many advantages such as good numerical stability, implementation with ease and satisfactory filtering accuracy as compared with other methods [1]. However, the existing third-degree ECKF still has some drawbacks: it may have low filtering accuracy when an unsuitable value of free parameter is chosen, and it is difficult to choose a suitable value in practice. Furthermore, the filtering accuracy of the third-degree ECKF can be improved by utilizing higher degree ECR.

In this paper, to solve these problems, a new adaptive method is proposed to determine the value of free parameter of ECKF based on maximum likelihood criterion, incorporating which a new third-degree adaptive ECKF (AECKF) algorithm is obtained. To further improve the accuracy of the third-degree AECKF, a new fifth-degree AECKF based on the fifth-degree ECR is developed. Simulation results show that the proposed methods have higher estimation accuracy than existing Gaussian approximated (GA) filters [2–7].

The remainder of the paper is organized as follows. The problem of existing third-degree ECKF is described in Section 2. The proposed third-degree AECKF is introduced in Section 3. The proposed fifth-degree ECKF and fifth-degree AECKF are developed in Section 4. Simulation is given in Section 5. Concluding remarks are drawn in Section 6.

2. Problem of the existing third-degree ECKF

Consider the following discrete-time nonlinear stochastic dynamic system as shown by the state-space model [2]:

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{f}_{k-1}(\boldsymbol{x}_{k-1}) + \boldsymbol{w}_{k-1} \\ \boldsymbol{z}_k = \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{v}_k \end{cases} \tag{1}$$

where k is the discrete time index, $\boldsymbol{x}_k \in \mathbb{R}^n$ is the state vector, $\boldsymbol{z}_k \in \mathbb{R}^m$ is the measurement vector, $\boldsymbol{w}_k \in \mathbb{R}^n$ and $\boldsymbol{v}_k \in \mathbb{R}^m$ are uncorrelated zero-mean Gaussian white noise vectors satisfying $E[\boldsymbol{w}_k \boldsymbol{w}_l^T] = \boldsymbol{Q}_k \delta_{kl}$ and $E[\boldsymbol{v}_k \boldsymbol{v}_l^T] = \boldsymbol{R}_k \delta_{kl}$ respectively, where δ_{kl} is the Kronecker delta function, the initial state

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$\boldsymbol{x}_0$ is a Gaussian random vector with mean $\hat{\boldsymbol{x}}_{0|0}$ and covariance matrix $\boldsymbol{P}_{0|0}$, and it is uncorrelated with $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$.

The third-degree ECR can be formulated as [1]

$$\int_{\mathbb{R}^n} \boldsymbol{g}(\boldsymbol{x}) N(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{\Sigma})\, dx \approx \left(1-\frac{1}{2\delta^2}\right)\boldsymbol{g}[\boldsymbol{\mu}] + \frac{1}{2^{n+1}\delta^2}\sum_{\boldsymbol{s}} \boldsymbol{g}\left[\boldsymbol{\mu}+\sqrt{2\boldsymbol{\Sigma}}\boldsymbol{\delta}\right] \tag{2}$$

where $\boldsymbol{x}=[x_1,x_2,\ldots,x_n]^T$, and $\sqrt{\boldsymbol{\Sigma}}$ is the square-root of the covariance matrix $\boldsymbol{\Sigma}$, i.e. $\boldsymbol{\Sigma}=\sqrt{\boldsymbol{\Sigma}}\sqrt{\boldsymbol{\Sigma}}^T$, and $\boldsymbol{s}=(s_1,s_2,\ldots,s_n)$, and $\boldsymbol{\delta}=(s_1\delta,s_2\delta,\ldots,s_n\delta)$ with $s_i=\pm 1$, and δ is a free parameter.

Different values of δ may result in different estimation accuracy of ECKF because δ determines the approximated error of high-degree terms. It is shown in [1] that the third-degree ECKF can achieve good numerical stability and asymptotically fifth-degree filtering accuracy by choosing proper value of δ. However, the third-degree ECKF may have bad filtering performance when an unsuitable value of δ is chosen. An optimal value of δ cannot be analytically obtained, and it is difficult to be determined in practice. Therefore, an adaptive estimation method is proposed based on maximum likelihood criterion to determine the value of δ in next section, based on which a new third-degree AECKF algorithm is obtained.

3. Third-degree AECKF

We define that the optimal value of δ is a solution that maximizes the likelihood function of the measurement vector. Under the Gaussian assumption of likelihood function $p(\boldsymbol{z}_k|\boldsymbol{Z}_{k-1})=N(\boldsymbol{z}_k;\hat{\boldsymbol{z}}_{k|k-1},\boldsymbol{P}^{zz}_{k|k-1})$, where $\hat{\boldsymbol{z}}_{k|k-1}$ and $\boldsymbol{P}^{zz}_{k|k-1}$ denote the predicted measurement and innovation covariance matrix respectively, we have

$$p(\boldsymbol{z}_k|\boldsymbol{Z}_{k-1})=\frac{1}{\sqrt{\left|2\pi\boldsymbol{P}^{zz}_{k|k-1}\right|}}\exp\left(-\frac{1}{2}\|\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1}\|^2_{\boldsymbol{P}^{zz-1}_{k|k-1}}\right) \tag{3}$$

where $\|\boldsymbol{x}\|^2_{\boldsymbol{P}}=\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x}$. As the measurement $\boldsymbol{z}_k$ is available at time k, the likelihood function can always be computed after the one-step prediction estimate vector $\hat{\boldsymbol{z}}_{k|k-1}$ and covariance matrix $\boldsymbol{P}^{zz}_{k|k-1}$ of measurement have been obtained. Since both $\hat{\boldsymbol{z}}_{k|k-1}$ and $\boldsymbol{P}^{zz}_{k|k-1}$ are δ dependent in ECKF, the likelihood function (3) is also δ dependent, and can be rewritten in the following form to emphasize its dependence on δ:

$$p(\boldsymbol{z}_k|\boldsymbol{Z}_{k-1},\delta)=\frac{1}{\sqrt{\left|2\pi\boldsymbol{P}^{zz}_{k|k-1,\delta}\right|}}\exp\left(-\frac{1}{2}\|\boldsymbol{z}_k-\hat{\boldsymbol{z}}_{k|k-1,\delta}\|^2_{\boldsymbol{P}^{zz-1}_{k|k-1,\delta}}\right) \tag{4}$$

When the measurement $\boldsymbol{z}_k$ is available at time k, the estimate of δ achieving the highest likelihood at time k is chosen as the optimal free parameter and can be formulated as follows:

$$\hat{\delta}=\arg\max_{\delta} p(\boldsymbol{z}_k|\boldsymbol{Z}_{k-1},\delta) \tag{5}$$

In most cases, it is not possible to find a closed-form solution to (5) because δ is coupled in the complex nonlinear operation. Thus, it is necessary to use a numerical method to fulfill the above-mentioned optimization computation. In this paper, a numerical grid method is used to compute $\hat{\delta}$, and it covers feasible domain $[\delta_{min},\ \delta_{max}]$ with searching step length sl. This method has also been used in unscented Kalman filter (UKF) method to adaptively set the scaling parameter [7]. The likelihood function is evaluated in these equally spaced grid of points, and the point with the highest likelihood is chosen. We denote the proposed adaptive algorithm as ASOFR (adaptive setting of free parameter), and its detailed steps are summarized in Table 1. The third-degree AECKF can be obtained by inserting $\hat{\delta}$ in the implementation of ECKF in (2). For the proposed third-degree AECKF, its feasible domain can be set as $[\sqrt{1/2},\ \sqrt{3/2}]$, as suggested in [1], and its searching step length sl can be chosen in $[0.001, 0.01]$ according to our simulation experience.

The accuracy of numerical technique used to compute integrals in GA filter determines the filtering accuracy, which implies that the filtering accuracy of the third-degree AECKF can be further improved by increasing the degree of ECR. Next a new fifth-degree AECKF will be proposed to improve the filtering accuracy of the third-degree AECKF.

4. Fifth-degree ECKF and fifth-degree AECKF algorithms

The Gaussian weighted integrals in GA filter can be expressed as the following general form [2]:

$$\boldsymbol{I}[\boldsymbol{g}]=\pi^{-n/2}\int_{\mathbb{R}^n}\boldsymbol{g}(\boldsymbol{x})e^{-\boldsymbol{x}^T\boldsymbol{x}}\,d\boldsymbol{x} \tag{6}$$

Table 1
The detailed steps of the proposed ASOFR algorithm.

The proposed ASOFR algorithm
(1) Choose the searching step length sl and feasible domain $[\delta_{min},\delta_{max}]$.
(2) Set the initial values of free parameter $\delta=\delta_{min}$ and maximal likelihood function value $maxLH=0$.
(3) Repeat (until $\delta>\delta_{max}$) the following loop:
(a) Compute predicted state $\hat{\boldsymbol{x}}_{k\|k-1,\delta}$ and predicted error covariance matrix $\boldsymbol{P}_{k\|k-1,\delta}$ by using (1) and (2) and considering $\boldsymbol{\mu}$ as $\hat{\boldsymbol{x}}_{k-1\|k-1}$ and $\boldsymbol{\Sigma}$ as $\boldsymbol{P}_{k-1\|k-1}$.
(b) Compute predicted measurement $\hat{\boldsymbol{z}}_{k\|k-1,\delta}$ and innovation covariance matrix $\boldsymbol{P}^{zz}_{k\|k-1,\delta}$ by using (1) and (2) and considering $\boldsymbol{\mu}$ as $\hat{\boldsymbol{x}}_{k\|k-1,\delta}$ and $\boldsymbol{\Sigma}$ as $\boldsymbol{P}_{k\|k-1,\delta}$.
(c) Compute the likelihood function value $LH=p(\boldsymbol{z}_k\|\boldsymbol{Z}_{k-1},\delta)$ through (4).
(d) If $LH\geq maxLH$, then $maxLH=LH$ and $\hat{\delta}=\delta$.
(e) $\delta=\delta+sl$.
(4) $\hat{\delta}$ is the adaptive setting value of free parameter δ.