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Fast communication

Fine resolution frequency estimation from three DFT samples: Case of windowed data

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ABSTRACT

An efficient and low complexity frequency estimation method based on the discrete Fourier transform (DFT) samples is described. The suggested method can operate with an arbitrary window function in the absence or presence of zero-padding. The frequency estimation performance of the suggested method is shown to follow the Cramer–Rao bound closely without any error floor due to estimator bias, even at exceptionally high signal-to-noise-ratio (SNR) values.

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1. Introduction

Frequency estimation of complex exponential signals is a fundamentally important non-linear parameter estimation problem arising in several applications. Recently, an efficient frequency estimation technique based on the samples of Discrete Fourier Transform (DFT) has been proposed in [1,2]. An important restriction of this technique and several others such as [3–7] is the requirement of DFT calculation with the rectangular window without any zero-padding. The present work aims to remove both of these restrictions by adapting the bias correction factor in [2] to the window of interest.

The frequency estimation method given in [1,2] consists of two stages. In the first stage (coarse frequency estimation), N-point Discrete Fourier Transform (DFT) of the N-point input is calculated. In the second stage (fine frequency estimation), the DFT bin with the maximum magnitude (k_p) and its immediate left (k_p-1) and right neighbors (k_p+1) are used to estimate the fine part of the frequency:

$$\hat{\delta} = c_N \ \mathrm{Real}\left\{ \frac{R[k_p-1]-R[k_p+1]}{2R[k_p]-R[k_p-1]-R[k_p+1]} \right\}. \tag{1}$$

Here c_N is $\frac{\tan(\pi/N)}{\pi/N}$ for the rectangular window. The final frequency estimate is formed by combining the results of both stages, $\hat{\omega} = 2\pi(k_p+\hat{\delta})/N$ radians/sample. The first stage of this estimator works with the rectangular window in the absence of zero-padding. Further details on this method can be found in [1,2].

In many applications, the DFT calculation is implemented with a properly selected window to suppress the interference caused by undesired spectrum components [8–10]. For example, in pulse Doppler radars, the desired signal (target echo at a specific Doppler frequency) coexists with other echos such as clutter signal, undesired target echos and jamming signal. With the application of windowing, the impact of interfering components on the desired signal is reduced. Due to emergence of the same problem in many applications, a number of frequency estimation methods with windowed data are given in the instrumentation and measurement literature [11–14]. A particularly well known estimator is the one utilizing Rife–Vincent class-I windows

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is [12,13]

$$\hat{\delta} = (M+1)\frac{|R_w[k_p+1]| - |R_w[k_p-1]|}{2|R_w[k_p]| + |R_w[k_p-1]| + |R_w[k_p+1]|}. \quad (2)$$

Here $|R_w[k_p]|$ is the peak value of the *windowed* DFT output where the applied window is the Rife–Vincent class-I window having the parameter $M = \{0, 1, 2, \ldots\}$. Rife–Vincent class-I windows are equivalent to the rectangular and Hann windows for $M=0$ and $M=1$, respectively. Due to the widespread usage of Hann window $M=1$ case of this estimator is important for many applications.

In an analogy with (1), the window specific correction factor for the estimator in (2) is $(M+1)$. It should be clear that the estimator (2) and its correction factor are specific to a particular window. To the best of our knowledge, apart from Duda's work [14], all other estimators in the literature are also derived for specific windows [9–11]. In [14], Duda presents a novel approach based on compensating the window specific estimator bias through a high order polynomial interpolation. The approach presented in this paper is very similar, in principle, to the one of Duda's. Here, we only adapt the bias compensation factor c_N in (1) to the window. The main advantage of the proposed method is its improved performance in spite of its low computational complexity.

2. Preliminaries

A complex exponential signal with the normalized frequency f in $[0, 1)$ and with the complex amplitude A is observed under additive white Gaussian noise:

$$r[n] = Ae^{j(2\pi fn + \phi)} + v[n], \quad n = \{0, \ldots, N-1\}. \quad (3)$$

The frequency f can also be denoted in terms of the DFT bins, $f = (k_p + \delta)/N$ where k_p is an integer in $[0, N-1]$ and δ is a real number in $-1/2 < \delta < 1/2$ [2]. The noise $v[n]$ is circularly symmetric complex valued Gaussian noise with zero mean and σ_v^2 variance, $v[n] \sim \mathcal{CN}(0, \sigma_v^2)$. The signal-to-noise ratio is defined as $\text{SNR} = A^2/\sigma_v^2$.

In many applications, the complex exponential signal is observed in the presence of interfering signals. For such applications, the DFT is calculated with a proper a window function to reduce the interference on the frequency estimate. Fig. 1 shows the spectrum with the Hamming window. The Hamming window with its low side-lobes reduces the interference leakage at the cost main-lobe widening [8, Chapter 6]. It can be also seen that the curvature around the peak changes significantly with the applied window. This is the main reason that an interpolation based frequency estimation method for a specific window does not work for any other window.

As noted in the introduction section, the first stage of the method described in [1] calculates the N-point DFT of $r[n]$ and then a peak search in the magnitude spectrum is conducted. This stage aims to estimate the coarse part of the frequency (k_p) as shown in Fig. 1. In the second stage, the fractional part of the frequency (δ) is estimated. For the rectangular windowed signal, δ is estimated through the relation (1) with $c_N = \frac{\tan(\pi/N)}{\pi/N}$. Our goal is to use the same relation for δ estimation, but select the bias-correction coefficient c_N according to the window function.

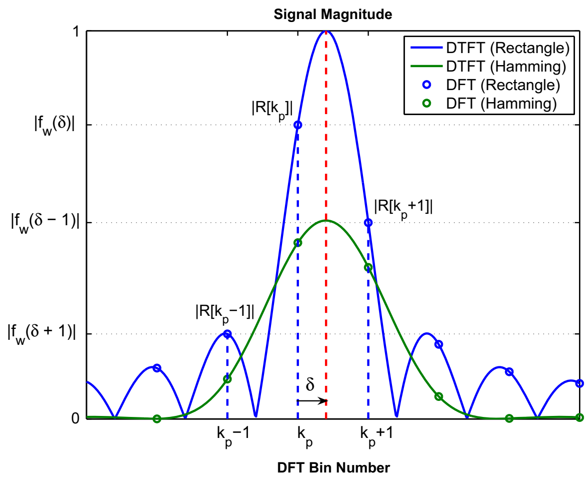

Fig. 1. DTFT and DFT spectrum of the complex exponential waveform with the frequency $k_p + \delta$ bins using rectangle and Hamming windows.

3. Main results

In the first stage of the proposed method, the input signal is transformed to the DFT domain after the application of the real valued window $w[n]$,

$$R[k] = \sum_{n=0}^{N-1} w[n]r[n]e^{-j(2\pi/N_2)kn}, \quad k = \{0, 1, 2, \ldots, N_2 - 1\} \quad (4)$$

where N_2 is the number of DFT points, which is possibly larger than N with the application of zero-padding. In the absence of noise, we may take, without any loss generality, $A=1$ in (3) and write $r[n]$ as $r[n] = e^{j2\pi(k_p+\delta)/N_2}$. Then, the spectrum samples $R[k_p + l]$ (l: integer) can be written as follows:

$$R[k_p + l] = \sum_{n=0}^{N-1} w[n]e^{j(2\pi n/N_2)(\delta + l)} = f_w(\delta + l). \quad (5)$$

The window dependent $f_w(\alpha)$ function appearing on the right hand side of (5) is explicitly defined as

$$f_w(\alpha) = \sum_{n=0}^{N-1} w[n]e^{j(2\pi n N_2)\alpha}. \quad (6)$$

For the case of zero-padding $(N_2 > N)$, the window function $w[n]$ in (6) can be considered as the zero-padded version of N-point window and $f_w(\alpha)$ for integer valued α values is its N_2-point inverse DFT.

3.1. Bias correction factor

An estimate for δ can be produced through the processing of the samples $R[k_p-1] = f_w(\delta+1), R[k_p] = f_w(\delta)$ and $R[k_p+1] = f_w(\delta-1)$ via the relation (1), provided that c_N is made available. Our goal is to set the bias correction factor c_N according the windowing function.

To facilitate the calculation of c_N, we expand the function $f_w(\delta + l)$ into the Taylor series around $\alpha = l$:

$$f_w(\delta + l) = f_w(l) + f'_w(l)\delta + O(\delta^2) \quad (7)$$

Here $f'_w(\alpha) = \frac{j2\pi}{N_2}\sum_{n=0}^{N-1} nw[n]e^{j(2\pi n/N_2)\alpha}$ is the first derivative of the function given in (6). Upon the substitution of

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