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# Nonlocal linear minimum mean square error methods for denoising MRI

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ABSTRACT

The presence of noise results in quality deterioration of magnetic resonance (MR) images and thus limits the visual inspection and influence the quantitative measurements from the data. In this work, an efficient two stage linear minimum mean square error (LMMSE) method is proposed for the enhancement of magnitude MR images in which data in the presence of noise follows a Rician distribution. The conventional Rician LMMSE estimator determines a closed-form analytical solution to the aforementioned inverse problem. Even-though computationally efficient, this approach fails to take advantage of data redundancy in the 3D MR data and hence leads to a suboptimal filtering performance. Motivated by this observation, we put forward the concept of nonlocal implementation with LMMSE estimation method. To select appropriate samples for the nonlocal version of the LMMSE estimation, the similarity weights are computed using Euclidean distance between either the gray level values in the spatial domain or the coefficients in the transformed domain. Assuming that the signal dependent component of the noise is optimally suppressed by this filtering and the rest is a white and uncorrelated noise with the image, we adopt a second stage LMMSE filtering in the principal component analysis (PCA) domain to further enhance the image and the noise variance is adaptively adjusted. Experiments on both simulated and real data show that the proposed filters have excellent filtering performance over other state-of-the-art methods.



## 1. Introduction

Magnetic resonance imaging (MRI) is an invaluable diagnostic tool and an essential noninvasive imaging modality that provides a vast amount of anatomical and functional information useful for diagnosis and patient treatment. It is often the case that the noise in the magnitude MR images follows Rician distribution when acquired with single coil. Consideration of how noise affects the true signal is important for proper interpretation and analysis of MR images [1].

Noise filtering plays an important role in the enhancement of MR images. A plethora of different denoising methods have been proposed in last two decades [2]. Many authors directly applied traditional smoothing filters and conventional classical denoising techniques to treat the noise in MR images with an assumption of the Gaussian distributed noise model. However, those attempts have been failed to minimize the bias due to Rician noise. The bias becomes particularly important in low SNR MR images and it increases with decreasing SNR.

As a solution to the aforementioned problem, numerous methods have been proposed in the literatures that can be mainly classified into those based on partial differential equations (PDE) [3–8], wavelet based methods [9–11], nonlocal means (NLM) [12–16] or nonlocal maximum likelihood (NLML) methods [17–21]. Wavelet-based filters are rooted in the processing of images in a transformed domain. Bao and Zhang [22] reduced the noise in MR images based on an adaptive multiscale products threshold which incorporates the merits of interscale dependencies into the thresholding technique for denoising. Other transforms that have been applied to denoise images include principal component analysis (PCA) [23] and discrete cosine transform (DCT) [24]. Many transform domain filters have derived based on the transform-threshold-inverse transform principles. An adaptive diffusion method for magnitude MR data was proposed by Sijbers et al. [4]. Also, the authors in [5] proposed a noise adaptive nonlinear diffusion technique to denoise MR images with spatially varying noise

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levels. The limitations of the above mentioned techniques are that they usually tend to remove the useful high frequency details of the image and generate unnatural structures due to the undesirable estimations at edges.

Apart from the above discussed methods, another category of methods has been proposed for MR denoising that relied on the statistical estimation theorem. Sijbers et al. [25,26] estimated the noise variance from Rician distributed MR data and carried out signal estimation using a maximum likelihood (ML) method [19,27]. Other customized versions of the ML method have been addressed in [17,28,29]. He et al. proposed a nonlocal ML (NLML) estimation method to overcome the disadvantages of local ML method such as blurring of edges and the distortion of fine structures in the image. In that method, the samples for the ML estimation are selected in a nonlocal way based on the intensity similarity measurement of the pixel neighborhoods using the Euclidean distance. In 2014, Rajan et al. [20] presented a new NLML estimation method, for noise reduction in MR images that follows Rician distribution, in which the samples are selected in an adaptive and statistically supported way using the Kolmogorov–Smirnov test. Wong et al. [30] proposed a novel stochastic noise reduction method for MR data which utilizes Quasi-Monte Carlo estimation (QMCE) approach for estimating the noise-free signal. The QMCE approach learns the statistical characteristics of the underlying noise distribution, as well as taking into account the regional statistics of the observed signal, in a data-adaptive manner.

Recently, Aja-Fernández et al. [31,32] suggested a computationally efficient noise-driven anisotropic diffusion filtering based on a closed-form Rician LMMSE estimator for the large 3D MR images. This method estimates the noiseless signal value using local statistics of the observed image contents, i.e., by selecting a set of pixels from a local neighborhood. So, the high frequency components of the image like edges and fine details where the different underlying gray levels in a local neighborhood lead to the different realizations of its Rician nature; as a result of which, adverse estimation will be obtained. In an attempt to avoid the problem, Sudeep et al. [33] proposed a hybrid algorithm by incorporating the goodness of LMMSE estimation approach with Split-Bregman TV denoising method through 3D wavelet-subband mixing method to improve the quality of the denoised image. Since the MR data intrinsically contain many similar samples (patches) that can be used to improve the estimation results, Golshen et al. [34] addressed this difficulty by developing an SNR adapted nonlocal LMMSE method that takes the advantage of the high degree of redundancy in the contents of MR images and using a similarity measure based on the local statistical moments of the image. In [35], a filter based on nonlocal neutrosophic set (NLNS) approach was proposed. Filter bank based nonlocal means [36], iterative bilateral filtering [37], MR denoising in the wavelet packet transformed domain [38] are the other latest trends of the research area.

In this paper we proposed an improved LMMSE method for denoising magnitude MR images in which the data (in the presence of noise) follows Rician distribution. The rest of the paper is structured as follows: Section 2 presents the relevant background on the noise characteristics in MRI. Section 3 elaborates the proposed method. Section 4 deals with the experimental results, comparative evaluation and discussion, followed by the conclusions and remarks in Section 5.

## 2. Noise in MRI

### 2.1. Noise characteristics in MRI

The real and imaginary part of the raw complex valued MRI data, with mean values $A_R$ and $A_I$ respectively, are corrupted by white Gaussian noise with variance $\sigma_n^2$. Since the computation of magnitude image, as the root of the sum of squares (SoS) of the real and imaginary part of the complex signal, is a nonlinear operation, the distribution of the observed magnitude MR data with noise will be Rician distributed and is given by [39]:

$$p_M(M|A, \sigma_n) = \frac{M}{\sigma_n^2} e^{-(M^2+A^2/2\sigma_n^2)} I_0\left(\frac{AM}{\sigma_n^2}\right) H(M) \tag{1}$$

where $I_0$ is the 0th order modified Bessel function of the first kind. Here, $M$ denotes the Rician distributed random variable, $A = \sqrt{(A_R^2 + A_I^2)}$ and $H(\cdot)$ represents the Heaviside step function. The shape of the Rician distribution depends on the signal to noise ratio (SNR), which is here defined as the ratio $A/\sigma_n$. At high SNR, i.e., when $A/\sigma_n \to \infty$, the Rician distribution approaches a Gaussian distribution and its probability density function (PDF) can be written as [39]:

$$p_M(M, \sigma_n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-((M-A)^2/2\sigma_n^2)} H(M) \tag{2}$$

In the image background, where $A$ equals zero, the Rician PDF simplifies to Rayleigh distribution [40]:

$$p_M(M, \sigma_n) = p_M(M|A = 0, \sigma_n) = \frac{M}{\sigma_n^2} e^{-(M^2/2\sigma_n^2)} H(M) \tag{3}$$

### 2.2. Noise estimation

Although all information is contained in the real and imaginary parts of the complex data acquired by an MRI system, the usual output of the scanners are magnitude images. The straight forward and most reliable approach to estimate noise from the magnitude images is to use a double acquisition method. When two images of the same subject are acquired under identical imaging conditions, noise variance can be estimated using the averaged and single images. Many methods have been proposed in the literature to estimate the noise from magnitude MR images [41]. A survey of those methods are given in [42]. However most of the methods proposed in the literature exploit the Rayleigh distributed background region for noise estimation. These methods may not work properly when the background is very less. Recently some object based methods are proposed in literature [43,44] and these methods doesn't depends on the background for noise estimation. In this paper, we followed [44] for estimating the noise from the MR images.

## 3. Theory and method

### 3.1. Signal estimation using LMMSE estimator

The most extensively used estimator to obtain a closed-form solution for a signal that obeys a Rician PDF is the LMMSE estimator. Closed form estimation methods are more efficient with computations than optimization based solutions like maximum likelihood (ML) and expectation maximization (EM) techniques. Aja-Fernández et al. have shown that $\mathbf{A}^2$ instead of $\mathbf{A}$ can be used to achieve a closed form expression whereby all moments to be used will be even. The LMMSE estimator for Rician distributed data, that is simplified for point wise estimation, is defined as [34]:

$$\begin{aligned}\hat{A}_{i,j,k}^2 &= \langle M_{i,j,k}^2\rangle - 2\sigma_n^2 + (M_{i,j,k}^2 - \langle M_{i,j,k}^2\rangle) \\ &\quad\times \max\left(1 - \frac{4\sigma_n^2(\langle M_{i,j,k}^2\rangle - \sigma_n^2)}{\langle M_{i,j,k}^4\rangle - \langle M_{i,j,k}^2\rangle}, 0\right)\end{aligned} \tag{4}$$