Signal Processing 100 (2014) 85–92

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Unbiased estimation of Markov jump systems with distributed delays

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ARTICLE INFO

Article history:
Received 17 April 2013
Received in revised form
29 October 2013
Accepted 29 December 2013
Available online 8 January 2014

Keywords:
Markov jump systems (MJSs)
Unbiased H_∞ filtering
Distributed time-delays
Linear matrix inequality

ABSTRACT

The unbiased H_∞ filtering problem is considered for a class of Markov jump systems (MJSs) with distributed time-delays. Based on the selected Lyapunov–Krasovskii functional, it gives a sufficient condition for the existence of the mode-dependent unbiased H_∞ filter such that the filtering error dynamic MJSs is stochastically stable and satisfies a prescribed level of H_∞ disturbance attenuation in an infinite time-interval. The design criterions are presented in the form of linear matrix inequality techniques, and then are described as the optimization problems. At last, two numerical examples are employed to illustrate the effectiveness of the developed techniques.

1. Introduction

In the past several decades, dynamical systems with distributed time-delays have received considerate attentions. Many efforts have been made to study the stability and control problems of this kind of systems. Among these results, an important and popular approach is to construct a Lyapunov–Krasovskii function for the distributed time-delays. Then by using the bonding techniques to the cross-term, delay-dependent or delay-independent criteria are obtained. Comparing with the delay-independent criteria, the delay-dependent ones are less conservative conditions for the stability analysis and controller synthesis. And the measuring index is the maximum allowable upper bound in the delays, for instance [1–6]. But for filtering schemes, the conservative conditions are seldom considered. The filtering problem is to estimate the unavailable state variables of a known system by using past measurement. During the past several years, H_∞ filtering problems have been investigated extensively. By comparison with the traditional Kalman filtering scheme [7], the noise sources in the H_∞ setting are arbitrary signals with bounded energy or average power, and no exact statistical properties are required to be known. For more results on this topic, readers are referred to Refs. [8–12]. It should point out that the dimensions of the filtering error system, which is constructed by the dynamics and the filter, are twice that of the dynamics. This may bring the computational complexity, especially in practical engineering process where the number of states is numerous. For these, the so-called unbiased conditions [13] can provide a good help. Under this condition, the dimension of the unbiased filtering error dynamics equals to that of the dynamic system. Moreover, no restriction is imposed on the stability of original dynamics while taking into account the unbiased condition. So this filtering approach has extensive application in unstable systems. For more results on this topic, we refer readers to [14–19] and the references therein.

In general, the parameters of many dynamical systems are subject to random abrupt changes due to, for example, sudden environment changes, subsystem switching, system

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http://dx.doi.org/10.1016/j.sigpro.2013.12.030

noises, failures occurred in components or interconnections and executor faults, etc. Markov jump systems (MJSs), as a special kind of hybrid systems with two components, the mode and the state, can be employed to model the above phenomena. The dynamics of jump modes and continuous states in MJSs are respectively modeled by finite state Markov chains and differential equations. With two dynamical mechanisms, time-evolving and event-driven, the applications of MJSs are more comprehensive since the pioneering work of Krasovskii and Lidskii on quadratic control [20] in the early 1960s. The existing results about MJSs cover a large variety of problems such as stochastic stability [21–25] stochastic controllability [26–28] and the references therein. In recent years, the filtering schemes for MJSs have gained a great deal of attention, and a large number of results [29–38] are also available. It is worth noticing that towards each case above, more details are related to the regular H_∞ filtering problems and the system time-delay is the retarded one which only contains time-delay in its states. In practice, with respect to modeling errors, unknown inputs and distributed time-delayed dynamic systems may be more reasonable to account for the realistic process. This motives our research of this topic.

In this paper, we consider the design problem of unbiased H_∞ filtering for a class of distributed time-delayed MJSs. We first construct the full-order linear H_∞ filter with distributed time-delays of original MJSs. Subsequently, under the unbiased filtering condition, the dynamics of filtering error are developed. Then, in order to guarantee the robustness against disturbances, the H_∞ filtering problem is formulated to minimize the influences of the unknown disturbances. Furthermore, by using the constructed Lyapunov–Krasovskii functional and linear matrix inequalities (LMIs) [39] approach, the sufficient condition on the solution of unbiased H_∞ filter are presented and proved. Finally, the unbiased H_∞ filtering problem is formulated as an optimization algorithm. Simulation results illustrate the effectiveness of the developed techniques.

In the sequel, if not explicitly states, matrices are assumed to have compatible dimensions. The notations used throughout this paper are quite standard. The symbols $\mathfrak{R}^n$ and $\mathfrak{R}^{n\times m}$ stand for an n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. A^{T} and A^{-1} denote the matrix transpose and matrix inverse. $diag\left\{A \quad B\right\}$ represents the block-diagonal matrix of A and B. $\sigma_{min}(*)$, $\sigma_{max}(*)$ mean the minimum and the maximum eigenvalue of the corresponding matrices. $\mathbf{E}\{*\}$ stands for the mathematics statistical expectation of the stochastic process or vector and $\|*\|$ is the Euclidean vector norm. $L_2^n[0,+\infty)$ is the space of n-dimensional square integrable function vector over $[0,+\infty)$. $P>0$ stands for a positive-definite matrix. I is the unit matrix with appropriate dimensions. 0 is the zero matrix with appropriate dimensions. In symmetric block matrices, we use "$*$" as an ellipsis for the terms that are introduced by symmetry.

2. System description

Given a probability space (Ω, F, P_r) where Ω is the sample space, F is the algebra of events and P_r is the probability measure defined on F. Let us consider a class of linear distributed time-delayed MJSs defined in the probability space (Ω, F, P_r) and described by the following differential equations:

$$\begin{cases} \dot{x}(t)-A_3(\delta_t)\dot{x}(t-\tau)=A(\delta_t)x(t)+A_1(\delta_t)x(t-h)+A_2(\delta_t)\int_{t-\eta}^{t}x(s)ds+B(\delta_t)w(t) \\ y(t)=C_y(\delta_t)x(t)+D_y(\delta_t)w(t) \\ z(t)=C_z(\delta_t)x(t)+D_z(\delta_t)w(t) \\ x(t)=\sigma(t),\ \delta_t=\xi(t),\ t\in\left[t_0-H \quad t_0\right] \end{cases} \tag{1}$$

where $x(v)\in\mathfrak{R}^n$ is the state, $y(t)\in\mathfrak{R}^l$ is the measured output, $w(t)\in L_2^m\left[0 \quad +\infty\right)$is the unknown disturbance, $z(t)\in\mathfrak{R}^q$ is the controlled output, $\sigma(t)$ is a vector-valued initial continuous function defined on the interval $\left[t_0-H \quad t_0\right]$ and $\xi(t)$ is the initial mode., the time delays $h>0$, $\eta>0$ and $\tau>0$ are assumed to be known and $H=max\left\{h \quad \eta \quad \tau\right\}$. $A(\delta_t)$, $A_1(\delta_t)$, $A_2(\delta_t)$, $A_3(\delta_t)$, $B(\delta_t)$, $C_y(\delta_t)$, $D_y(\delta_t)$, $C_z(\delta_t)$, $D_z(\delta_t)$ are known mode-dependent constant matrices with appropriate dimensions.

The jump parameter δ_t in (1) represents a continuous-time discrete-state Markov stochastic process taking values on a finite set $\mathbf{M}=\{1,2,\ldots,N\}$ with transition rate matrix $\Pi=\{\pi_{ij}\}$, $i,j\in\mathbf{M}$ and has the following transition probability from mode i at time t to mode j at time $t+\Delta t$ as

$$P_{ij}=P_r\{\delta_{t+\Delta t}=j|\delta_t=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j\end{cases} \tag{2}$$

where $\Delta t>0$ and $\lim_{\Delta t\downarrow 0}o(\Delta t)/\Delta t\to 0$. $\pi_{ij}\geq 0$ is the transition probability rates from mode i at time t to mode $j(i\neq j)$ at time $t+\Delta t$, and$\sum_{j=1,j\neq i}^{N}\pi_{ij}=-\pi_{ii}$.

Remark 1. To simplify the study, we assume that the time-delays in (1) are constant and only dependent of the system structure, and they are not dependent on the defined stochastic process. Furthermore, we will take the initial time $t_0=0$ and let the initial values $\left\{\sigma(t)\right\}_{t\in\left[-d \quad 0\right]}$ and $\left\{\xi(t)=\delta_t\right\}_{t\in\left[-d \quad 0\right]}$ be fixed. At each mode, we assume that the time-delay MJSs have the same dimension. For convenience, when $\delta_t=i$, $A(\delta_t)$, $A_1(\delta_t)$, $A_2(\delta_t)$, $A_3(\delta_t)$, $B(\delta_t)$, $C_y(\delta_t)$, $D_y(\delta_t)$, $C_z(\delta_t)$, $D_z(\delta_t)$ are respectively denoted as A_i, A_{1i}, A_{2i}, A_{3i}, B_i, C_{yi}, D_{yi}, C_{zi}, D_{zi}.

Remark 2. For the relevant estimation problems of the distributed time-delayed MJSs (1), the matrices parameters A_{2i}, A_{3i}, satisfy $A_{2i}\neq 0$, $A_{3i}\neq 0$. In order to guarantee the stability of the differential operator $\chi(x_t)=x(t)-A_{3i}x(t-\tau)$, we always set $\|A_{3i}\|<1$, where $\|*\|$ is the relevant matrix norm of every modes. In fact, Markov jump systems with mixed delays (constant, varying, distributed or neutral time-delays) have received some attention and several research results have been proposed [1–6].

In this paper, our aim is to develop techniques of unbiased H_∞ filtering problem for time-delay MJSs (1) and to obtain an estimate $\hat{z}(t)$ of the signal $z(t)$ such that the defined index performance is minimized in an unbiased estimation error sense. Thus, we construct the