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Face-image retrieval based on singular values and potential-field representation

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ABSTRACT

In this paper, an efficient method based on singular values and potential-field representation is proposed for face-image retrieval. Firstly, we theoretically prove that the leading singular values of an image can be used as a rotation-shift-scale-invariant global feature. Then, for the feature-extraction stage, we exploit these special properties of the singular values to devise a compact, global feature for face-image representation. We also use the singular values of the potential field derived from edge gradients to enhance the retrieval performance. Experimental results based on the GTAV database show that the use of singular values as rotation-shift-scale-invariant global features is able to produce plausible retrieval results.

1. Introduction

Content-based face-image retrieval has been an active research area in recent years because of its many practical applications, including social networks [1], object tracking [2], surveillance systems [3], and forensic scenarios [4], etc. Many face-image retrieval algorithms have been proposed recently. A face matching and retrieval method using soft biometrics, such as facial marks, scars, and demographics was proposed in [3]. Gao and Qi [5] proposed a visual similarity measuring technique to retrieve face images in photo-album databases for law enforcement. In [6], a scalable face-image retrieval system was proposed, which employs a scalable face representation using both local and global features. Later, Smith et al. [7] proposed a new face-search technique based on shape manipulation to address the problem of finding faces with slightly more furrowed brows or specific leftward-pose shifts.

In the past decades, many efficient and discriminative features have been proposed for face recognition [8–11]. However, these features are typically high-dimensional and global, and are thus not suitable for quantization or inverted indexing [6]. Consequently, using such global features for image retrieval essentially requires a linear scan of the whole database in order to process a query, which is prohibitive for a web-scale image database [6]. Singular value decomposition (SVD) is an effective technique for image retrieval. Hsu and Chen [16] proposed a SVD-based method for face recognition, which can be viewed as a two-sided 2DPCA method. A feature-extraction and image-retrieval algorithm using the Discrete Wavelet Transform (DWT) and SVD was proposed in [17]. In that method, SVD is used to extract a feature of each area from an area-division model. A matching method based on SVD was proposed in [18]. The singular matrix is calculated based on the SVD transformation, and is mapped into a vector space; then image similarity is measured by comparing the angle between vectors. In [19], Cao and Yang proposed an improved face recognition algorithm based on SVD, which uses a sampling window to sample images with overlapping from left to right and

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top to bottom, and then utilizes the fist k maximum singular values of each matrix to form a group of vectors. In [20], Lu and Zhao proposed a dominant SVD representation (DSVDR) method, which aims to select a subset of important bases and regulate their singular values according to their discriminative and reconstructive power, for face recognition. To reduce the computational complexities of a content-based image-retrieval (CBIR) system, an image is first decomposed by SVD, and only the leading singular values are then used to reconstruct the image, which is used instead of the original image by the CBIR system [21]. Later, using the eigenvectors corresponding to the k largest eigenvalues, a new subspace of dimension k is built to represent images in [22]. Liu et al. [26] proposed a novel fractional-order SVD representation (FSVDR) method for face recognition. More recently, a new face recognition method based on difference vectors plus KPCA (DVKPCA) was proposed in [27].

In contrast to the above-mentioned SVD-based methods, in this paper we propose an efficient face-image retrieval method based on singular values and potential-field representation. To the best of our knowledge, this is the first paper which completely proves that singular values can be used as a rotation-shift-scale-invariant global image feature based on the Frobenius norm. In this paper, we will first prove that the leading singular values of an image can be used as a compact rotation-shift-scale-invariant global face representation. In addition, the edge-gradient images of faces can be used for face recognition. In order to enhance the retrieval performance, the singular values of potential-field representations derived from edge gradients [12,13] are also incorporated in our proposed method.

The rest of the paper is organized as follows. In Section 2, we will present Singular Value Decomposition and the rotation-shift-scale-invariant features. Section 3 introduces potential-field representation, and Section 4 proposes the combined similarity function. Experimental results are presented in Section 5. The paper closes with a conclusion and discussion in Section 6.

2. SVD-based rotation-shift-scale-invariant feature

2.1. Singular value decomposition

In this section, we use a mathematical framework to present an effective image representation [23]. Our proposed framework projects an image M_I of size $m \times n$ to an eigenspace using SVD. The image M_I can be viewed as a matrix with m rows and n columns. Assuming that $m \geq n$, by using SVD, M_I can be written as the product of a left matrix U, a $n \times n$ diagonal matrix W with positive or zero diagonal elements, and the transpose of a right matrix V, i.e.

$$M_I = UWV^T, \tag{1}$$

where $U^T U = V^T V = I$ and I is an identity matrix. The matrix U is a $m \times m$ column-orthogonal matrix, while V is a $n \times n$ orthogonal matrix. The elements w_i on the diagonal of W are arranged in descending order, and are called the singular values (the square root of the eigenvalues) of M_I, i.e.

$$W = \mathrm{diag}(w_1, w_2, ..., w_i ...., w_n). \tag{2}$$

The k leading singular-value vector s of the image M_I is defined as follows:

$$s = [w_1, w_2, ..., w_k]^T, \tag{3}$$

where $1 \leq k \leq n$, and w_i is the ith singular value of M_I in the singular-value vector s such that $w_i \geq w_{i+1}$. It can be observed that the singular values decrease dramatically and the first few leading eigenvectors can account for most of the information. Thus, we have

$$\sum_{i=1}^{n} {w_i}^2 \doteq \sum_{i=1}^{k} {w_i}^2, \tag{4}$$

where k is the number of singular values or eigenvectors to be retained.

2.2. The rotation-shift-scale-invariant feature

According to [24], if a matrix A has singular values w_i, where $1 \leq i \leq n$, then

$$||A||_F = \sqrt{\sum_{i=1}^{n} {w_i}^2}, \tag{5}$$

where $||A||_F$ is the Frobenius norm of the matrix A, which is defined as the square root of the sum of the squares of all its entries. The following is a brief proof of (5):

Proof. If matrix A has singular values, $w_1, w_2, ...,$ $w_i ....,$ w_n, then $||A||_F^2 = \sum_{i=1}^{n} {w_i}^2$.

Using SVD, a matrix A can be expressed as $A = UWV^T$. First note that, for any matrix C whose ith column is denoted as c_i, i.e. $C = (c_1|...|c_n)$, then $||C||_F^2 = ||c_1||_F^2 + ... + ||c_n||_F^2$. Now, we have

$$\begin{aligned} ||A||_F^2 &= ||UWV^T||_F^2 \\ &\overset{[1]}{=} ||WV^T||_F^2 = ||(WV^T)^T||_F^2 = ||(V^T)^T W^T||_F^2 = ||VW^T||_F^2 \\ &\overset{[2]}{=} ||W^T||_F^2 = ||W||_F^2 \overset{[3]}{=} \sum_{i=1}^{n} {w_i}^2 \end{aligned}$$

[1] Let $(c_1|...|c_n)$ be the columns of WV^T. Since the matrix U simply rotates the columns of $(c_1|...|c_n)$ without changing their lengths, the two sides are equal.

[2] The matrix V simply rotates the columns of W_T without changing their lengths.

[3] The diagonal matrix W has positive or zero values on its diagonal, and the other elements are all zeros.□

2.2.1. Rotation-invariant property

Theorem 1. *If an image I_a is rotated to produce a new image I_b, then the first k main singular values in the singular-value vector s_b of the new image I_b are equal to the corresponding first k main singular values of the singular-value vector s_a of the original image I_a (i.e. $s_a = s_b$).*

Proof. Without loss of generality, suppose that I_a is rotated by an angle β to produce a new image I_b, then