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Multi-frequency inversion in Rayleigh damped Magnetic Resonance Elastography

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ABSTRACT

Magnetic Resonance Elastography (MRE) is able to identify mechanical properties of biological tissues *in vivo* based on underlying assumptions of the model used for inversion. Models, such as the linearly elastic or viscoelastic (VE), can be used with a single input frequency data and can produce a reasonable estimate of identified parameters associated with mechanical properties. However, more complex models, such as the Rayleigh damping (RD) model, are not identifiable given single frequency data without significant *a priori* information under certain conditions, thus limiting diagnostic potential. To overcome this limitation, two approaches have been postulated: simultaneous inversion across multiple input frequencies and a parametric approach, when only single frequency data is available.

This research compares simultaneous multi-frequency (MF) RD reconstructions using both zero-order and power-law (PL) models with parametric reconstructions for a series of tissue-simulating phantoms, made of tofu and gelatine materials, tested at 4 frequencies (50 Hz, 75 Hz, 100 Hz and 125 Hz) that are commonly applied in clinical MRE examinations. Results indicate that accurate delineation of RD based properties and concomitant damping ratio (ξ_d) using MF inversion is still a challenging task. Specific results showed that the real shear modulus (μ_R) can be reconstructed well, while imaginary components representing attenuation (μ_I and ρ_I) had much lower quality. However, overall trends correlate well with the expected higher damping levels within the saturated tofu material compared to stiff gelatine in both phantoms. Depending on the phantom configuration, measured μ_R values within the tofu and gelatine materials ranged from 4.77 to 7 kPa and 15.5 to 16.3 kPa, respectively, while damping levels were 11–19% and 3.1–4.3%, as expected. Correlation of the μ_R and ξ_d values with previously reported result measured by independent mechanical testing and VE based MRE is acceptable, ranging from 48 to 60%. Both PL and zero-order models produced similar qualitative and quantitate results, thus no significant advantage of the PL model was noted to account for dispersion characteristics of these types of materials.

The relatively narrow range of frequencies used in this study limited practical identifiability and can thus produce a potentially false assurance of identifiability of the model parameters. We conclude that application of multiple input frequencies over a wide range, as well as selection of an appropriate model that can accurately account for dispersion characteristics of given materials are required for achieving robust practical identifiability of the RD model in time-harmonic MRE.

1. Introduction

In time-harmonic Magnetic Resonance Elastography (MRE), a single input frequency is a common experimental approach for mechanical property reconstruction. It is less time consuming and only requires a single patient examination. A number of studies investigated various models for both static [1,2] and dynamic [3–11] MRE methods using single frequency data. These models include linearly elastic [12–14], viscoelastic (VE) [15–20] and even poroelastic [21–23] models.

Two main limitations prevent single frequency approaches from providing accurate approximations of the tissue response. The first is associated with the dispersive nature of the complex shear modulus at different frequencies and thus quantitative estimates acquired at a particular excitation frequency do not represent the true static (at 0 Hz) values of material constants. The second arises

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from complex boundary conditions and undesirable wave interaction between propagating and reflected waves inside the tissue. This effect can cause amplitude nulls where no elasticity information is available. The first issue can be mitigated by simultaneous inversion of multiple wave data sets acquired at different excitation frequencies and finding an appropriate model that can accurately account for dispersion characteristics and associated frequency dependent behaviour [24]. However, the second issue presents challenging limitations to current inversion methods.

Thus, a multi-frequency (MF) approach offers a number of benefits over a single frequency approach. First, it allows dispersion measurement of the rheological tissue parameters by analysing multiple propagating wave velocities corresponding to different driving frequencies. This can enable more accurate measurement of the viscosity which might be potentially useful in tissue characterisation. Second, it can avoid wave amplitude nulls accruing in single-frequency wave patterns. Previous MF elastography studies found a correlation between biological tissue response to various excitation frequencies in a form of power-law (PL). A PL implies frequency dependant behaviour of the VE parameters.

More complex models, such as the *Rayleigh* or proportional damping (RD) model [25,26], capture this frequency dependant behaviour. However, they are not identifiable with single frequency data [27]. Hence, multiple actuation frequencies are required for more complex models, as well. Previous studies of MF applications have mainly utilized two parameter (Voigt model, Maxwell model) and three-parameter (Zener model, Jeffreys model) VE models [28–31], while the RD model has been previously discussed in the context of simulation studies for a homogeneous material [32,33]. Preliminary single-frequency RD MRE experiments compared reconstruction results for the RD and linear VE models with mixed results [34]. However, Petrov et al. [27] concluded that the RD model was not uniquely identifiable for single frequency data. Without a guarantee of *a priori* identifiability, the parameter estimates obtained might be unreliable or random. Two alternative approaches are suggested to overcome non-identifiability of the RD model: 1. simultaneous MF inversion and 2. parametric inversion, when only single frequency data is available.

Consequently, a MF approach might contribute towards theoretical identifiability of the RD model [27] and is expected to produce more robust, data driven results than possible with a single frequency data. This study evaluates MF inversion in RD MRE in comparison to a parametric approach to accurately delineate the RD parameters and assess the efficacy of MF for RD reconstructions. Two alternative material models are also exploited, a zero-order model and a PL model.

2. Materials and methods

2.1. RD model in time-harmonic MRE

The RD model is implemented through Finite Element (FE) based solution of a nearly-incompressible linear isotropic Navier's equation, defined:

$$\nabla\cdot(\mu(\nabla\mathbf{u}+\nabla\mathbf{u}^T))-\nabla(\lambda\nabla\cdot\mathbf{u})-\nabla P=-\rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

where $\mathbf{u}$ is the displacement within the medium; λ is the first Lamé's parameter ($\lambda = 1/3$ for a nearly-incompressible case), μ is the second Lamé's parameter, also known as the shear stiffness; ρ is the density of the material, ∇P is a pressure term, related to volumetric changes through the bulk modulus, K, via the relationship: $\nabla P = K\nabla\cdot\mathbf{u}$.

It can be discretised and defined:

$$\mathbf{M}\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\mathbf{K}\mathbf{u}=\mathbf{f}, \tag{2}$$

for the mass, damping and stiffness matrices, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, respectively; displacement vector, $\mathbf{u}$, and known sinusoidal input forcing, $\mathbf{f}$. The RD assumption is facilitated through the RD definition:

$$\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}, \tag{3}$$

where α and β are proportionality constants, $\alpha > 0$ and $\beta > 0$. For a time-harmonic case, where input and resulting response are $\mathbf{f}(\mathbf{x},t)=\hat{\mathbf{f}}(\mathbf{x})e^{i\omega t}$ and $\mathbf{u}(\mathbf{x},t)=\hat{\mathbf{u}}(\mathbf{x})e^{iwt}$, Eq. (2) in the frequency domain is written:

$$[-\omega^2(1-\frac{i\alpha}{\omega})\mathbf{M}+(1+i\omega\beta)\mathbf{K}]\,\hat{\mathbf{u}}=\hat{\mathbf{f}}. \tag{4}$$

By assuming stiffness and density to be complex valued, e.g. $\mu^* = \mu_R + i\mu_I$ and $\rho^* = \rho_R + i\rho_I$, Eq. (4) can be further simplified:

$$[-\omega^2\rho^*\mathbf{M}'+\mu^*\mathbf{K}']\,\hat{\mathbf{u}}=\hat{\mathbf{f}}, \tag{5}$$

where $\mathbf{M}'$ and $\mathbf{K}'$ are normalised mass and stiffness matrices, respectively, defined:

$$\mathbf{M}'=\frac{1}{\rho}\begin{pmatrix}1&0&\cdots&0\\0&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1\end{pmatrix},\qquad \mathbf{K}'=\frac{1}{\mu}\begin{pmatrix}\omega_1^2&0&\cdots&0\\0&\omega_2^2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&\omega_n^2\end{pmatrix}. \tag{6}$$

μ_R and ρ_R describe the real valued shear modulus and density in the undamped system, while μ_I and ρ_I represent two different damping components related to elastic and inertial effects, respectively; and can be expressed in terms of the RD parameters:

$$\rho_I=\frac{-\alpha\rho_R}{\omega},\qquad \mu_I=\omega\beta\mu_R. \tag{7}$$

The resulting damping ratio, ξ_d, is defined:

$$\xi_d=\frac{1}{2}\left(\beta\omega+\frac{\alpha}{\omega}\right)\ \Rightarrow\ \xi_d=\frac{1}{2}\left(\frac{\mu_I}{\mu_R}-\frac{\rho_I}{\rho_R}\right). \tag{8}$$

Eq. (8) indicates that the stiffness proportional term ($\beta\omega$) contributes damping linearly proportional to the response frequency and the mass proportional term (α/ω) contributes damping inversely proportional to the response frequency. Therefore, the α and β coefficients govern the response to lower and higher input frequencies, ω, respectively.

The RD model offers promising potential to better characterise complex damping behaviour in highly saturated medias as it accounts for an additional damping mechanism (ρ_I) arising from viscous effects. Those effects are typically observed in saturated structures due to high fluid content (commonly seen in biological tissues), and are associated with viscous attenuation arising from the inertial forces.

2.2. Structural non-identifiability of the RD model

Collecting real and imaginary terms from Eq. (5), yields:

$$[(\mu_R-\omega^2\rho_R)+i(\mu_I-\rho_I)]\,\hat{\mathbf{u}}=\hat{\mathbf{f}}. \tag{9}$$

Assuming a known ρ_R value, Eq. (9) implies that μ_R is uniquely identifiable as a direct function of the real displacements: $\mu_R = \mathbf{f}_R/\mathbf{u}_R + \mu^2\rho_R$. However, the model behaviour dictated by the imaginary part of the $\hat{\mathbf{u}}$ coefficient is determined by two model parameters, ρ_I and μ_I, which are both variables to be identified. Hence, both parameters have a model role, but there cannot be unique identification of those variables at one frequency without further *a priori* information.