Signal Processing 92 (2012) 2874–2880

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Stability of 2-D digital filters described by the Roesser model using any combination of quantization and overflow nonlinearities

Priyanka Kokil, Anurita Dey, Haranath Kar *

Department of Electronics and Communication Engineering, Motilal Nehru National Institute of Technology, Allahabad 211004, India

ARTICLE INFO

Article history:
Received 31 January 2012
Received in revised form
14 May 2012
Accepted 15 May 2012
Available online 23 May 2012

Keywords:
Asymptotic stability
Digital filter
Finite wordlength effect
Lyapunov method
Nonlinear system

ABSTRACT

A novel linear matrix inequality (LMI)-based criterion for the global asymptotic stability of two-dimensional (2-D) state-space digital filters described by the Roesser model employing various combinations of quantization and overflow nonlinearities is presented. The criterion is less restrictive than a previously reported criterion.

1. Introduction

Two-dimensional (2-D) systems find applications in many areas such as filtering, image processing, modeling of partial differential equations, medical imaging, face recognition, geophysics, projective radiography, thermal processes in chemical reactors, 2-D digital control systems, river pollution modeling, process of gas filtration, grid based wireless sensor networks, etc. [1–7]. The 2-D systems are characterized by two independent variables propagating information in two independent directions. Due to a broad range of applications of 2-D discrete systems, there has emerged a continuously growing interest in the system theoretic problems of such systems.

Filtering is one of the fundamental problems in the control and signal processing areas. The problem of L_2-L_∞ filter design has been studied for continuous time-delay stochastic systems [8] and for Takagi-Sugeno (T-S) fuzzy discrete-time systems with time-varying delay [9]. An elegant approach for the stability analysis and stabilization of discrete-time T-S fuzzy systems with time-varying state delay has been presented in Ref. [10]. Ref. [11] investigates the robust H_∞ filtering problem for a class of linear fractional uncertain continuous-time nonlinear systems with interval time-varying delays. A parameter-dependent approach to robust H_∞ filtering for uncertain discrete-time systems with polytopic uncertainties has been presented in Ref. [12]. The H_∞ filtering problem for a class of discrete-time networked nonlinear systems with mixed random delays and packet dropouts has been solved in Ref. [13]. In Ref. [14], the problem of H_∞ model approximation for discrete-time T-S fuzzy time delay systems has been tackled. A method for H_∞ model reduction of T-S fuzzy stochastic systems has been given in Ref. [15].

In recent years, a great number of fundamental notions and results of one-dimensional (1-D) discrete systems have been generalized to 2-D systems. Ref. [16] deals with the problem of H_∞ filtering for 2-D discrete system in Roesser model [17] with Markovian jump parameters.

* Corresponding author. Tel.: +91 532 2271815; fax: +91 532 2545341.
E-mail addresses: kokilnit@gmail.com (P. Kokil), anurita.johorey@rediffmail.com (A. Dey), hnkar1@rediffmail.com (H. Kar).

0165-1684/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sigpro.2012.05.016

The problem of robust H_∞ filter design for uncertain 2-D Fornasini–Marchesini second model [18] with parameter uncertainties and missing measurements has been investigated in Ref. [19]. The robust H_∞ control problem for uncertain 2-D discrete delayed systems in the general model via non-fragile state feedback controllers has been addressed in Ref. [20].

When 2-D digital filters are implemented in finite wordlength processors using fixed-point arithmetic, nonlinearities are introduced owing to the quantization and overflow. The presence of such nonlinearities may result in instability of the designed filter [21–24]. The global asymptotic stability of the null solution guarantees the absence of limit cycles in the realized digital filters. The finite wordlength effects in digital filters depend on the kind of quantization and overflow nonlinearities employed, as well as the kind of arithmetic involved. If the number of quantization step is large or, in other words, the internal wordlength is sufficiently long, then the effects of quantization and overflow can be treated as decoupled or mutually independent. Under the decoupling approximation, a number of researchers have extensively investigated quantization effects of 2-D digital filters in the absence of overflow [25–28], while others have studied the effect of overflow in the absence of quantization effects [29–42]. Nevertheless, the validity of decoupling assumption has been queried by several researchers [43–45]. The study of digital filters involving both quantization and overflow nonlinearities is considered to be more realistic because the real digital filter operates in the presence of both quantization and overflow nonlinearities. A few publications relating to the issue of stability of such digital filters have appeared in the literature [46–49]. The approach in Ref. [47] is based on the determination of stability regions where overflow cannot occur. In particular, Ref. [47] presents a criterion under which the global asymptotic stability of a system involving quantization and overflow nonlinearities is shown to be equivalent to the global asymptotic stability of that with only quantization nonlinearities. Ref. [48] brings out improved versions of some of the stability results given in Ref. [47]. An alternative approach for stability analysis of 1-D and 2-D digital filters under various combinations of quantization and overflow nonlinearities has also been presented in Ref. [48]. The approach in Ref. [48] is based on the sector information of the finite wordlength nonlinearities where overflow nonlinearities are assumed for every state in the system. However, when dealing with the systems with finite wordlength nonlinearities, one often encounters situations where all the states do not reach to the overflow level. Thus, there still remains scope for characterizing the finite wordlength nonlinearities precisely in order to achieve enhanced stability region in the parameter-space, as compared to that obtainable via Refs. [47,48].

In this paper, we study the problem of global asymptotic stability of fixed-point 2-D digital filters described by Roesser model [17] under finite wordlength nonlinearities. The paper is organized as follows: Section 2 presents a description of the system under consideration and previously reported criteria. In Section 3, we establish a new criterion for the global asymptotic stability of 2-D digital filters under various combinations of quantization and overflow nonlinearities and for the situation where quantization occurs after summation only. The criterion makes use of a more precise sector-based characterization of the finite wordlength nonlinearities than the characterization used in Ref. [48]. A comparison of the proposed criterion with Refs. [47,48] is made in Section 4. In particular, it is demonstrated that the proposed criterion is an improvement over Refs. [47,48]. Finally, Section 5 provides the conclusion.

2. System description and previously reported criteria

The following notations are used throughout the paper:

$R^{p\times q}$	set of $p\times q$ real matrices
R^p	set of $p\times 1$ real vectors
$\mathbf{0}$	null matrix or null vector of appropriate dimension
$\boldsymbol{I}_p$	$p\times p$ identity matrix
Z_+	set of nonnegative integers
$\boldsymbol{B}^T$	transpose of the matrix (or vector) $\boldsymbol{B}$
$\boldsymbol{B}>\mathbf{0}$	$\boldsymbol{B}$ is positive definite symmetric matrix
$\boldsymbol{B}=\boldsymbol{B}_1\oplus\boldsymbol{B}_2$	direct sum, i.e., $\boldsymbol{B}=\boldsymbol{B}_1\oplus\boldsymbol{B}_2=\begin{bmatrix}\boldsymbol{B}_1 & \mathbf{0}\\ \mathbf{0} & \boldsymbol{B}_2\end{bmatrix}$
$\|\|\cdot\|\|$	any vector or matrix norm
$\|\|\cdot\|\|_\infty$	maximum absolute row sum norm

The system under consideration is the zero-input 2-D discrete systems described by the Roesser model [17] under the influence of various combinations of quantization and overflow nonlinearities. Specifically, consider the state-space quarter-plane model given by

$$\boldsymbol{x}_{11}(k,l)=\begin{bmatrix}\boldsymbol{x}^h(k+1,l)\\ \boldsymbol{x}^v(k,l+1)\end{bmatrix}=\boldsymbol{O}\{\boldsymbol{Q}(\boldsymbol{y}(k,l))\}=\boldsymbol{f}(\boldsymbol{y}(k,l))=\begin{bmatrix}\boldsymbol{f}^h(\boldsymbol{y}^h(k,l))\\ \boldsymbol{f}^v(\boldsymbol{y}^v(k,l))\end{bmatrix}, \tag{1a}$$

$$\boldsymbol{y}(k,l)=\begin{bmatrix}\boldsymbol{y}^h(k,l)\\ \boldsymbol{y}^v(k,l)\end{bmatrix}=\begin{bmatrix} y_1^h(k,l) & y_2^h(k,l) & \cdots & y_m^h(k,l) & y_1^v(k,l) & y_2^v(k,l) & \cdots & y_n^v(k,l)\end{bmatrix}^T=\begin{bmatrix}\boldsymbol{A}_{11} & \boldsymbol{A}_{12}\\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}^h(k,l)\\ \boldsymbol{x}^v(k,l)\end{bmatrix}=\boldsymbol{A}\boldsymbol{x}(k,l), \tag{1b}$$

$$\boldsymbol{f}^h(\boldsymbol{y}^h(k,l))=\begin{bmatrix} f_1^h(y_1^h(k,l)) & f_2^h(y_2^h(k,l)) & \ldots & f_m^h(y_m^h(k,l))\end{bmatrix}^T, \tag{1c}$$

$$\boldsymbol{f}^v(\boldsymbol{y}^v(k,l))=\begin{bmatrix} f_1^v(y_1^v(k,l)) & f_2^v(y_2^v(k,l)) & \ldots & f_n^v(y_n^v(k,l))\end{bmatrix}^T, \tag{1d}$$

$$k\ge 0,\quad l\ge 0, \tag{1e}$$

where $k\in Z_+$ and $l\in Z_+$ are the space coordinates. The state vectors $\boldsymbol{x}^h(k,l)\in R^m$ and $\boldsymbol{x}^v(k,l)\in R^n$ convey information horizontally and vertically, respectively. The $\boldsymbol{A}_{11}\in R^{m\times m}$, $\boldsymbol{A}_{12}\in R^{m\times n}$, $\boldsymbol{A}_{21}\in R^{n\times m}$, $\boldsymbol{A}_{22}\in R^{n\times n}$ are the