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Tsallis entropy and the long-range correlation in image thresholding

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ABSTRACT

Thresholding methods based on entropy have been proposed and developed over the years. In this paper, an improved Tsallis entropy based thresholding method is proposed for segmenting the images which presenting local long-range correlation rather than global long-range correlation. The advantage of the proposed method is to distinguish the pixels' local long-range correlation by the nonextensive parameter q. And the experimental results of various infrared images as well as nondestructive test ones show the effectiveness of the proposed method.

1. Introduction

Image segmentation is very important in image process. The purpose of segmentation is to distinguish and extract object regions from their background. One widely used method is to extract images by dividing brightness and/or gray levels, which is called "thresholding". The basic assumption of this method is that the objects and background have distinguishable gray-level distributions. Many different techniques of thresholding have been proposed and developed over the years [1–18]. A common idea among these methods is optimizing some threshold dependent functions which include the information and properties of the images. For instance, Kapur et al. considered the image's gray-level histogram as a kind of probability distribution and then one can obtain a t-dependent posteriori entropy of the image, where t is the gray-level threshold that maximize the entropy of the histogram [1]. If the neighborhood correlation between the pixels of an image is taken into account, a kind of two-dimensional entropy is proposed for thresholding the gray-level images [10,11]. Furthermore, the traditional entropic form based on Shannon theory can be generalized by an additional nonextensive parameter and it is named "Tsallis entropy" [19]. This new kind of entropy has been used into the image thresholding techniques and many novel results were obtained [7,12,13]. In fact, the concept of nonextensive entropy is firstly proposed to deal with the physical systems presenting long-range interaction and/or long-duration memory. The nonextensive parameter is correlated with the complex internal interactions within the system. For the digital images, we can also consider that the intensities of pixels have long-range correlations. Take a picture which has clear bilevel structure for example, it is reasonable to assume that there exist long-range correlations among the pixels belonging to the object (or background). The areas of the object (or background) localized the range of the correlation so it can be called as local long-range correlation. However, it is inadequate to say that there are long-range correlations between the object and background, especially for a series of images taken from the same background. In order to distinguish this kind of local long-range correlation from the global long-range correlation, a new technique of image thresholding based on Tsallis entropy is proposed in the present paper. The rest of this paper is organized as follows: Section 2 is a brief review

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about the Tsallis entropy and the nonextensive parameter q; Section 3 proposes a new criterion for segmenting the images which have obvious local long-range correlation rather than global long-range correlation; Section 4 gives some testing results of this technique and Section 5 draws the conclusions.

2. Tsallis entropy and nonextensive parameter *q*

From a physics point of view, the concept of entropy can be traced from the second law of thermodynamic which illustrates the irreversibility of processes in the universe. The irreversibility of a process in a system can be measured by the distribution of microstates of such system. Suppose that each microstate has a probability p_i and then the thermodynamical Boltzmann–Gibbs entropy is given by [20]

$$S=-k\sum_{i=1}^{W} p_i \ln p_i, \tag{1}$$

where k is the Boltzmann constant and W is the total number of microstates of the system. The Euler's number e is adopted as the base of logarithm in Eq. (1). In information theory, entropy is a measure of uncertainty of a random variable which has a probability distribution in a set of possible values. It is Shannon [21] that firstly redefined Eq. (1) for the measure of information. Here p_i is considered as the probability of a random variable at its ith possible value so we have $p_i \geq 0$.

In fact, Eq. (1) was proposed without considering the interactions among particles in the system. The microstates of the system can be considered as unit volumes in the phase space and they are mutually independent. Similarly, this ideal assumption means that each possible value for the random variable is independent, which is widely accepted nowadays in the field of information theory. However, with the development of modern physics, it has been observed in more and more systems that the internal interactions, especially for long-range interactions, are very important for the behavior of the systems [22–24]. Tsallis entropy is proposed under this background and it can be written as [19]

$$S_q=\frac{1-\sum_{i=1}^{W} p_i^q}{q-1}, \tag{2}$$

where q is a real number to describe the nonextensivity of the systems. For the sake of convenience, we set the Boltzmann constant $k=1$. One can easily verify that Eq. (2) will reduce to the traditional Boltzmann–Gibbs–Shannon entropy in the $q \to 1$ limit. By this evidence, it is Albuquerque et al. [12] who firstly proposed Tsallis entropy for digital image thresholding. And the basic assumption of this idea is that the pixels among an image may have long-range correlations in the intensity (gray-level). The nonextensive parameter q is effective to evaluate this kind of correlation and the thresholding result looks better than that of Shannon entropy.

3. The proposed method

Albuquerque's thresholding approach is reviewed in this section, and its limitation is discussed. A new threshold criterion also based on the entropy concept is then proposed.

3.1. Albuquerque's thresholding approach

Let $p_1,p_2,\ldots,p_n$ be the observed probability distribution of the gray-level histogram of an image. Thus, two probability distributions can be derived from this distribution, one for the background (class A) and the other for the object (class B). They can be written as

$$\text{A}: \quad \frac{p_1}{P_A},\frac{p_2}{P_A},\ldots,\frac{p_t}{P_A} \tag{3}$$

$$\text{B}: \quad \frac{p_{t+1}}{P_B},\frac{p_{t+2}}{P_B},\ldots,\frac{p_n}{P_B} \tag{4}$$

where $P_A=\sum_{i=1}^{t} p_i$ and $P_B=\sum_{i=t+1}^{n} p_i$.

Substituting Eqs. (3) and (4) into Eq. (2) yields:

$$H_q^A(t)=\frac{1-\sum_{i=1}^{t}(p_i/P_A)^q}{q-1} \tag{5}$$

$$H_q^B(t)=\frac{1-\sum_{i=t+1}^{n}(p_i/P_B)^q}{q-1} \tag{6}$$

The Tsallis entropy for the observed probability distribution is formulated as the sum of each entropy, allowing the pseudo-additive property for the subsystems. It is given by

$$\begin{aligned}H_q^{A+B}(t)&=H_q^A(t)+H_q^B(t)+(1-q)H_q^A(t)H_q^B(t)\\&=\frac{1-\sum_{i=1}^{t}(p_i/P_A)^q}{q-1}+\frac{1-\sum_{i=t+1}^{n}(p_i/P_B)^q}{q-1}\\&\quad+(1-q)\frac{1-\sum_{i=1}^{t}(p_i/P_A)^q}{q-1}\,\frac{1-\sum_{i=t+1}^{n}(p_i/P_B)^q}{q-1}\end{aligned} \tag{7}$$

From Eq. (7) we can see that the objective function is parametrically dependent upon the threshold value t that separates the foreground and background. Thus, the optimal threshold t_{opt} is $t_{opt}=Arg\,\max[H_q^{A+B}(t)]$. It indicated that some probabilities of gray-level in the histogram are not independent of each other so the information measure of them should be different from the Shannon entropy. The difference between Tsallis entropy and Shannon entropy depends on the nonextensive parameter q and it will vanish when $q \to 1$. Therefore, q represents the strength of the long-range correlation. The objective function of Eq. (7) means that there exist global long-range correlation in the areas of object and background, but also between the object and background. And the strength of the global correlation is described by the same q value. However, for some images with different correlation strength in the object and background, one should employ two different q values to describe them, respectively.

3.2. New criterion

Albuquerque's method is recognized as an effective threshold selection approach for general real-world images.