Signal Processing 92 (2012) 3010–3025

Contents lists available at SciVerse ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

# Robust reliable dissipative filtering for discrete delay singular systems ☆

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A R T I C L E  I N F O

*Article history:*
Received 12 March 2012
Received in revised form
15 May 2012
Accepted 2 June 2012
Available online 15 June 2012

*Keywords:*
Dissipative filtering
Reliable filtering
Singular systems
Time delay

A B S T R A C T

This paper is concerned with the problem of robust reliable dissipative filtering for uncertain discrete-time singular system with interval time-varying delay and sensor failures. The uncertainty and the sensor failures considered are polytopic uncertainty and varying in a given interval, respectively. The purpose is to design a filter such that the filtering error singular system is regular, causal, asymptotically stable and strictly $(Q,S,R)$-dissipative. By utilizing reciprocally convex approach, firstly, sufficient reliable dissipativity analysis condition is established in terms of linear matrix inequalities (LMIs) for singular systems with time-varying delay and sensor failures. Based on this criterion, the result is extended to uncertain singular systems with time-varying delay and sensor failures. Moreover, the reliable dissipative filter is designed in terms of linear matrix inequalities (LMIs) for uncertain singular systems with time-varying delay and sensor failures. Finally, the effectiveness of the filter design method in this paper is illustrated by numerical examples.



## 1. Introduction

During the past two decades, time delay systems have received considerable attention and have been extensively studied in the literature due to their widely application in various engineering systems and their bad effects on the performance and stability of the control systems [9,21,26,41,42]. Discrete-time systems with time-varying delay have more stronger application background compared with the corresponding continuous-time systems [6,11,16,30,31]. Many approaches have been proposed to develop delay-dependent conditions for discrete-time system with time-varying delay. To mention a few, Moon's inequality is used in [7]; new bounding inequalities and free-weighting matrix method are utilized in [6]; in order to construct new Lyapunov functionals, both $x(k-d_1)$ and $x(k-d_2)$ are employed in [44]; less conservative results are obtained by using convex combination approach in [25]. By using the model approximation method and the delay partitioning method, the proposed results in [30] and [31] are much less conservative than most of the existing results, respectively. In recent years, singular systems have also attracted growing attention due to the fact that such systems provide a more natural description of dynamic systems than the standard state-space systems. Singular systems can often preserve the structure of physical systems more accurately by including non-dynamic constraints and impulsive elements. Therefore, the stability and control problems of singular systems with time delay are investigated [33].

On the other hand, considerable attention has been devoted to the $H_\infty$ filtering problem due to its major theoretical significance for the decade [8,17,29], and a great many applications in the aerospace industry [23], TV tracking systems

☆ This work was partially supported by GRF HKU 7137/09E.

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0165-1684/$ - see front matter 
http://dx.doi.org/10.1016/j.sigpro.2012.06.003

[35] and data segmentation [32]. Moreover, the $H_\infty$ filtering results based on the theory of state-space systems have been successfully extended to singular systems. For continuous-time singular systems, full-order and reduced-order $H_\infty$ filtering problems are investigated without decomposing system matrices in [38,39], respectively. Delay-dependent $H_\infty$ filtering design method is given for singular systems with time-varying delay in [34]. For discrete-time singular systems, by using "normal" transformation and singular value decomposition (SVD) approach, the $H_\infty$ filtering problem is investigated in [14]. When time-varying delay appears, robust $H_\infty$ filter design method for singular systems with polytopic uncertainty is considered in [13] without decomposing the system matrices or requiring extra assumption on the systems. Although the results with $E=I$ in [13] improve the results in [8,46]. There still exists room to improve, which motivates the study in this paper.

In actual implementation, conventional filters for a multi-input–multi-output plant may lead to unsatisfactory performance due to the temporary failures on sensors which results in incomplete signal delivered to actuators. Therefore the reliable filter design problem have attracted an increasing research attention. By employing adaptive method, adaptive reliable $H_\infty$ filters are designed to compensate the sensor failure effects on systems in [40]. Benefiting from the delay-partitioning method, the problems of reliable $H_\infty$ filtering for discrete time-delay systems with randomly occurred nonlinearities and Markovian jump systems with partly unknown transition probabilities are solved in [19,18], respectively.

In addition, the dissipative theory introduced in [28] and subsequently generalized in [12] not only unifies the $H_\infty$ and positive real control theory but also provides a more flexible and less conservative robust control design as it allows a better trade-off between gain and phase performances. However, there are few results on dissipative filtering which is more general and unifies the $H_\infty$ and passive filtering. By using sector-nonlinearity modeling techniques, a $(Q,S,R)$-dissipative fuzzy filter is designed for a class of nonlinear systems rewritten by a T-S fuzzy model in [20]. A sufficient condition for dissipative filtering problem of linear discrete systems is proposed in terms of LMI in [15]. A non-fragile dissipative filtering problem for a class of nonlinear discrete-time systems with sector bounded nonlinearities is investigated in [43]. However, the important problem of reliable dissipative filter for discrete singular systems with time-varying delay and uncertainties remains to be considered.

In this paper we consider the problem of robust reliable dissipative filtering for discrete-time singular systems with polytopic uncertainties, time-varying delay and sensor failures. The filter is designed by reciprocally convex approach proposed in [22] such that the filtering error singular system is regular, causal, asymptotically stable and strictly $(Q,S,R)$-dissipative. For singular systems with time-varying delay and sensor failures, sufficient condition of reliable dissipative analysis is obtained in terms of LMIs. Then the result is extended to uncertain case by introducing some variables to decouple the Lyapunov matrices and the filtering error system matrices. Moreover, the desired filter for uncertain singular systems with time-varying delay and sensor failures is obtained by solving a set of LMIs. Numerical examples are given to illustrate the effectiveness of the presented results.

The rest of this paper is briefly outlined as follows. In Section 2, the problem of robust reliable dissipative filtering is formulated and some necessary definitions and lemmas are given. The robust reliable dissipativity analysis results for uncertain singular systems with time-varying delay and sensor failures are presented and the filter design method is proposed in Section 3. Illustrative examples are provided in Section 4 to show the effectiveness of our results. We conclude the paper in Section 5.

*Notation*: The notation used throughout the paper is standard. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and $P>0$ $(\geq 0)$ means that $P$ is real symmetric and positive definite (semi-definite); $I$ and 0 refer to the identity matrix and zero matrix with compatible dimensions; $\star$ stands for the symmetric terms in a symmetric matrix and sym$(A)$ is defined as $A+A^T$; $\bullet$ represents matrices that are not relevant with our discussion; $l_2$ is the space of square summable infinite vector sequences; for any real function $x,y \in l_2$, $M$, we define $\langle x,My \rangle_T = \sum_{k=0}^{T} x^T(k)y(k)$; $\|\cdot\|$ refers to the Euclidean vector norm. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

## 2. Problem statement

Consider a class of linear discrete-time singular systems with time-varying delay described by

$$\begin{cases} Ex(k+1)=Ax(k)+A_dx(k-d(k))+Bw(k) \\ y(k)=Cx(k)+C_dx(k-d(k))+Dw(k) \\ z(k)=Lx(k)+L_dx(k-d(k))+Gw(k) \\ x(k)=\phi(k), \quad k=-d_2,-d_2+1,\ldots,0 \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector; $y(k)\in\mathbb{R}^m$ is the measured output; $z(k)\in\mathbb{R}^p$ represents the signal to be estimated; $w(k)\in\mathbb{R}^l$ is assumed to be an arbitrary noise belonging to $l_2$ and $\phi(k)$ is a known given initial condition sequence; $d(k)$ is a time-varying delay satisfying

$$1 \leq d_1 \leq d(k) \leq d_2 < \infty, \quad k=1,2,\ldots \tag{2}$$

The system matrices $A$, $A_d$, $B$, $C$, $C_d$, $D$, $L$, $L_d$, and $G$ with appropriate dimension belong to a convex polytopic set

$$\chi := (A,A_d,B,C,C_d,D,L,L_d,G)\in\Omega \tag{3}$$