Photoacoustics 2 (2014) 21–32

Contents lists available at ScienceDirect

Photoacoustics

journal homepage: www.elsevier.com/locate/pacs

Research Article

Investigation of the far-field approximation for modeling a transducer's spatial impulse response in photoacoustic computed tomography

Kenji Mitsuhashi, Kun Wang, Mark A. Anastasio *

Department of Biomedical Engineering, Washington University in St. Louis, St. Louis, MO 63130, USA

ARTICLE INFO

Article history:
Received 6 September 2013
Received in revised form 31 October 2013
Accepted 7 November 2013

Keywords:
Photoacoustic computed tomography
Image reconstruction

ABSTRACT

When ultrasonic transducers with large detecting areas and/or compact measurement geometries are employed in photoacoustic computed tomography (PACT), the spatial resolution of reconstructed images can be significantly degraded. Our goal in this work is to clarify the domain of validity of the imaging model that mitigates such effects by use of a far-field approximation. Computer-simulation studies are described that demonstrate the far-field-based imaging model is highly accurate for a practical 3D PACT imaging geometry employed in an existing small animal imaging system. For use in special cases where the far-field approximation is violated, an extension of the far-field-based imaging model is proposed that divides the transducer face into a small number of rectangular patches that are each described accurately by use of the far-field approximation.

1. Introduction

Photoacoustic computed tomography (PACT), also known as optoacoustic tomography, is a hybrid computed imaging modality that combines the rich contrast of optical imaging methods with the deep penetration and high spatial resolution of ultrasound imaging methods [1–3]. In PACT, a short laser pulse is employed to irradiate an object and pressure waves are produced via the thermoacoustic effect and are subsequently measured outside the object by use of ultrasonic transducers. From these data, a PACT image reconstruction algorithm is employed to produce an image that depicts the spatially variant absorbed optical energy density within the object.

When ultrasonic transducers with large detecting areas and/or compact measurement geometries are employed in PACT, the spatial resolution of PACT images reconstructed by use of algorithms that assume point-like ultrasonic transducers can be significantly degraded [4,5]. To mitigate this effect, a description of the transducers' spatial impulse responses (SIRs) [6] can be incorporated into a discrete imaging model that approximately describes the action of the imaging system. In recent studies, algorithms based on such imaging models have been developed for reconstructing PACT images [7–10]. For three-dimensional (3D) PACT studies, these algorithms are generally optimization-based and iterative in nature [9]. Even when implemented on high-performance computing platforms [11], 3D PACT reconstruction algorithms that compensate for the SIR can be computationally burdensome.

For ultrasonic transducers that have flat circular or rectangular detection surfaces, a far-field approximation can result in a closed-form expression for the SIR [12]. It has been demonstrated that this allows for the construction of discrete PACT imaging models that have desirable computational characteristics [7]. In particular, based on the far-field SIR approximation, a 3D PACT imaging model was proposed and investigated for use in an optimization-based iterative image reconstruction method [7]. While the far-field-based imaging model possesses attractive computational characteristics that facilitate 3D iterative image reconstruction, there remains an important need to clarify its domain of validity within the context of practical 3D PACT imaging system configurations.

In this article, computer-simulation studies are described that confirm the far-field-based 3D PACT imaging model is highly accurate for a 3D PACT imaging geometry employed in an existing small animal imaging system [13]. We also demonstrate that when an ultra-compact imaging geometry is employed, use of this imaging model can result in image artifacts associated with the violation of the far-field approximation. For use in such cases, an extension of the far-field-based imaging model is proposed that divides the transducer face into a small number (e.g. 2×2) of

* Corresponding author. Tel.: +1 314 935 3637.
E-mail addresses: kenji.mitsuhashi@wustl.edu (K. Mitsuhashi), wangk@seas.wustl.edu (K. Wang), anastasio@seas.wustl.edu, anastasio@wustl.edu (M.A. Anastasio).

2213-5979
http://dx.doi.org/10.1016/j.pacs.2013.11.001

rectangular patches that are each described accurately by use of the far-field approximation. The performance of the far-field-based and patch-based algorithms are quantitatively investigated in terms of the accuracy of the reconstructed images. A preliminary investigation of the noise robustness of the far-field-based imaging models is also reported.

2. Background

Below we review the salient features of the imaging physics, discrete PACT imaging model, and reconstruction method that will be employed in our studies. The reader is referred to Refs. [3,7] for additional details.

2.1. Canonical imaging physics

In PACT, a pulsed laser source is used to irradiate an object, and the photoacoustic effect results in the generation of a pressure field $p(\mathbf{r}, t)$ [1,2], where $\mathbf{r} \in \mathbb{R}^3$ and t is the temporal coordinate. In this work, the to-be-imaged object and surrounding medium are assumed to have homogeneous and lossless acoustic properties. Additionally, the optical illumination of the object is assumed to be instantaneous, i.e. the laser pulse width is negligible. Under these conditions, the photoacoustic field $p(\mathbf{r}, t)$ satisfies the wave equation [1]:

$$\left[\frac{\partial^2}{\partial t^2} - c^2\Delta\right] p(\mathbf{r}, t) = 0, \tag{1}$$

subject to the initial conditions

$$p(\mathbf{r}, t)|_{t=0} = \frac{\beta c^2}{C_p} A(\mathbf{r}), \qquad \left.\frac{\partial p(\mathbf{r}, t)}{\partial t}\right|_{t=0} = 0, \tag{2}$$

where $A(\mathbf{r})$ is a compactly supported and bounded function that represents the absorbed optical energy density, Δ is the 3D Laplacian operator, and β, c, C_p are the thermal expansion coefficient, the speed of sound, and the isobaric specific heat, respectively.

The pressure field $p(\mathbf{r}, t)$ is assumed to be measured with flat ultrasonic transducer elements arranged on an arbitrary surface enclosing the object. The transducer elements are assumed to have no disruptive influence on the pressure field and therefore the background medium can effectively be assumed to have an infinite extent. The solution of Eq. (1) subject to Eq. (2) is given by [1]

$$p(\mathbf{r}, t) = \frac{\beta}{4\pi C_p} \int_V d^3\mathbf{r}' A(\mathbf{r}') \frac{\partial}{\partial t} \frac{\delta(t - \|\mathbf{r} - \mathbf{r}'\|/c)}{\|\mathbf{r} - \mathbf{r}'\|}, \tag{3}$$

where $V \subset \mathbb{R}^3$ contains the support of $A(\mathbf{r})$, $\delta(t)$ is the one-dimensional Dirac delta function, and $\|\cdot\|$ is the Euclidean norm. In a mathematical sense, the goal of PACT is to determine $A(\mathbf{r})$ from knowledge of $p(\mathbf{r}, t)$ on some measurement aperture outside the object.

2.2. Discrete imaging model

In practice, the photoacoustic pressure field $p(\mathbf{r}, t)$ is degraded by the response of the ultrasonic transducer and sampled during the measurement process. Consider that the ultrasonic transducers collect data at Q locations $\{\mathbf{r}_{0,q}\}_{q=0}^{Q-1}$ that are specified by the index $q = 0, \ldots, Q-1$ and K temporal samples, specified by the index $k = 0, \ldots, K-1$, are acquired at each location with a sampling interval ΔT. Let the vector $\mathbf{u} \in \mathbb{R}^{QK}$ denote a lexicographically ordered version of the sampled data. The notation $[\mathbf{u}]_{qK+k}$ will be employed to denote the $(qK+k)$th element of $\mathbf{u}$, which is related to the pre-sampled voltage signal at location q, $u_q(t)$, as $[\mathbf{u}]_{qK+k} := u_q(t)|_{t=k\Delta t}$. In this way, $[\mathbf{u}]_{qK+k}$ represents the kth temporal sample recorded by the transducer at location $\mathbf{r}_{0,q}$.

A continuous-to-discrete (C-D) imaging model [3,7] for PACT can be generally expressed as

$$[\mathbf{u}]_{qK+k} = h^e(t) *_t \frac{1}{\Omega_q} \int_{\Omega_q} d^2\mathbf{r}\; p(\mathbf{r}, t)|_{t=k\Delta T}, \tag{4}$$

where $p(\mathbf{r}, t)$ is determined by $A(\mathbf{r})$ via Eq. (3), the surface integral is over the detecting area of the qth transducer that is denoted by Ω_q, $h^e(t)$ denotes the acousto-electric impulse response (EIR) of the transducers, which is assumed to be the same for all transducers, and $*_t$ denotes a temporal convolution operation defined as

$$f(t) *_t g(t) := \int_{-\infty}^{\infty} d\tau\; f(\tau) g(t - \tau),$$

where f and g are arbitrary functions of t. Note that Eq. (4) is a C-D imaging model in the sense that it maps the function $A(\mathbf{r})$ to the finite-dimensional vector $\mathbf{u}$.

To obtain a discrete-to-discrete (D-D) imaging model for use with iterative image reconstruction algorithms, a finite-dimensional approximate representation of the object function $A(\mathbf{r})$ can be introduced as [7,9,14,15]

$$A^a(\mathbf{r}) = \sum_{n=0}^{N-1} [\boldsymbol{\theta}]_n \phi_n(\mathbf{r}), \tag{5}$$

where the superscript 'a' denotes that $A^a(\mathbf{r})$ is an approximation of $A(\mathbf{r})$, $\{\phi_n(\mathbf{r})\}_{n=0}^{N-1}$ represents a collection of expansion functions, and $\boldsymbol{\theta} \in \mathbb{R}^N$ is a vector of expansion coefficients. In this work, the expansion functions $\{\phi_n(\mathbf{r})\}_{n=0}^{N-1}$ will be chosen as uniform spherical voxels:

$$\phi_n(\mathbf{r}) = \begin{cases} 1, & \|\mathbf{r} - \mathbf{r}_n\| \le \epsilon, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where $\mathbf{r}_n$ denotes the nth voxel location and ϵ is the voxel radius. We assume that the voxels are non-overlapping. The imaging model described below, however, remains valid for any collection of radially symmetric expansion functions $\{\phi_n(\mathbf{r})\}_{n=0}^{N-1}$.

Let

$$\tilde{u}_q(f) := \int_{-\infty}^{\infty} dt\;\; u_q(t) e^{-i2\pi f t}$$

denote the temporal Fourier transform of the pre-sampled voltage signal $u_q(t)$ at location $\mathbf{r}_{0,q}$. In practice, the temporal frequency samples $\tilde{u}_q^a(f)|_{f=l\Delta f}$, $l = 0, \ldots, L-1$, can be estimated by computing the discrete Fourier transform of the measured samples of $u_q(t)$ with a frequency interval Δf. The vector $\tilde{\mathbf{u}}$ will denote a lexicographically ordered representation of the sampled temporal frequency data corresponding to all transducer locations, i.e. $[\tilde{\mathbf{u}}]_{qL+l} := \tilde{u}_q^a(f)|_{f=l\Delta f}$.

Consider the case where the detection surfaces of the ultrasonic transducers are rectangular and flat with area $a \times b$ (Fig. 1). A D-D imaging model can be expressed as [9]

$$\tilde{\mathbf{u}} \approx \mathbf{H}\boldsymbol{\theta}, \tag{7}$$

where $\mathbf{H}$ is the system matrix of dimension $QL \times N$, whose elements are defined as

$$[\mathbf{H}]_{qL+l,n} = \tilde{p}_0(f) \tilde{h}^e(f) \left.\frac{\tilde{h}_q^s(\mathbf{r}_n, f)}{ab}\right|_{f=l\Delta f}. \tag{8}$$

Here,

$$\tilde{p}_0(f) = -i\frac{\beta c^3}{C_p f}\left[\frac{\epsilon}{c}\cos\frac{2\pi f\epsilon}{c} - \frac{1}{2\pi f}\sin\frac{2\pi f\epsilon}{c}\right] \tag{9}$$