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# Min–max waveform design for MIMO radars under unknown correlation of the target scattering

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ABSTRACT

We address the problem of robust waveform design for MIMO radars when the target scattering covariance matrix is unknown at both the transmit and receive sides. Following a min–max approach, the code matrix is designed to minimize the worst-case cost under all possible target covariance matrices. Differently from previous works we include the choice of the codeword's length in the design problem. For a large class of cost functions, the min–max solution has a simple and intuitive structure which we explain. In particular, the codeword's length must be chosen as large as possible in order to provide the transmitter with the largest degrees of freedom. Examples illustrating the behavior of the min–max codes are provided for different case studies.



## 1. Introduction

Radar systems with multiple, isotropic antennas at both the transmit and receive sides may take advantage of angular diversity by illuminating the target under different aspect angles [1–9]. The MIMO architecture grants a number of degrees of freedom which can be exploited to enhance target detectability/identifiability: the key ingredient is the adoption of suitably encoded waveforms. Optimal waveform design amounts to finding a space-time code matrix which minimizes a given cost function under a transmit energy constraint. Any reasonable cost function should take into account the amount of energy back-scattered from the target towards each receive antenna, which necessitates some prior knowledge as to both the radar cross-section (RCS) and the degree of correlation among different angular looks. However, this information is *not* under the designer's control since it depends upon the range and the size of the prospective target. In such a situation, robust waveform design can be preferred, and the code-matrix can be chosen so as to minimize the worst-case cost under all possible values of the unknown parameters [10–14].

In this paper we address the problem of robust waveform design in multiple-input multiple-output (MIMO) radar systems. We assume no prior information about the statistical properties of the target RCS, neither at the transmitter nor at the receiver, and we include the choice of the codeword's length in the design problem. We focus on a min–max approach and consider a general cost function, encompassing many commonly adopted performance measures, such as the average signal-to-disturbance ratio, the linear minimum mean square error in estimating the target response, the mutual information between the received signal echoes and the target response, and the approximation of the miss probability in the high- and low-signal regimes when a generalized likelihood ratio test (GLRT) is adopted to solve the detection problem. Generalizing previous results in [13,14], we

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show that (1) the rank of the min–max code-matrix must be equal to the number of transmit antennas in order to prevent any target miss, (2) the left eigenvectors of the min–max code must match the least interfered directions in the signal space, and more energy must be allocated to more interfered directions in order to equalize their signal-to-disturbance ratios (SDR's), and (3) the codeword length must be maximized so as to enlarge the signal space. A number of examples are provided to illustrate the impact of the codeword length on performances of the proposed min–max design.

The remainder of the paper is organized as follows. In the next section, we introduce the signal model for the considered MIMO radar system, while in Section 3 we state and solve the problem of robust waveform design following a min–max approach. Section 3.2 discusses some commonly adopted cost functions which fall in the considered category. In Section 4 the numerical examples are presented, while concluding remarks are given in Section 5.

*Notation.* Column vectors and matrices are indicated through boldface lowercase and uppercase letters, respectively. If $x \in \mathbb{C}^n$, $x_i$ denotes its $i$-th entry, for $i=1,\dots,n$. $\mathrm{diag}\{a_1,\dots,a_n\} \in \mathbb{C}^{n\times n}$ is the diagonal matrix with $a_1,\dots,a_n$ on the main diagonal, $\boldsymbol{I}_m$ is the identity matrix of order $m$, and $\boldsymbol{0}_m$ is the all-zero, $m$-dimensional vector. $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose, respectively. $\mathcal{M}_n$ denotes the set of Hermitian, positive semi-definite matrices from $\mathbb{C}^{n\times n}$. $\boldsymbol{A}^{1/2}$ is the unique positive semi-definite square root of $\boldsymbol{A} \in \mathcal{M}_n$. $\{\lambda_i(\boldsymbol{A})\}_{i=1}^n$ is the set of eigenvalues of $\boldsymbol{A} \in \mathbb{C}^{n\times n}$; if $\boldsymbol{A}$ is Hermitian the eigenvalues are sorted in descending order, i.e., $\lambda_1(\boldsymbol{A}) \geq \cdots \geq \lambda_n(\boldsymbol{A})$, and $\lambda_{\max}(\boldsymbol{A}) = \lambda_1(\boldsymbol{A})$, $\lambda_{\min}(\boldsymbol{A}) = \lambda_n(\boldsymbol{A})$. $\mathrm{tr}(\boldsymbol{A})$ is the trace of $\boldsymbol{A} \in \mathbb{C}^{n\times n}$, while $\otimes$ denotes the Kronecker (tensor) product. A function $f : \mathcal{A} \subseteq \mathbb{R}^n \to \mathbb{R}$ is decreasing if for any $\boldsymbol{x},\boldsymbol{y} \in \mathcal{A}$, $f(\boldsymbol{x}) \geq f(\boldsymbol{y})$ whenever $x_i \leq y_i$, $\forall\, i=1,\dots,n$. Finally, $\mathbb{E}$ denotes statistical expectation.

## 2. Signal model

Consider an $M\times L$ MIMO radar whose task is to detect the presence of a target in a given range cell. Following the model of [4], the signal at the $\ell$-th receive antenna, $\ell=1,\dots,L$, can be expressed as

$$\boldsymbol{r}_\ell = \begin{cases} \boldsymbol{A}\boldsymbol{\alpha}_\ell + \boldsymbol{w}_\ell & \text{under } H_1 \\ \boldsymbol{w}_\ell & \text{under } H_0 \end{cases} \tag{1}$$

where

- $\boldsymbol{r}_\ell \in \mathbb{C}^N$;
- $N \leq \overline{N}$ is the signal space dimension;
- $\overline{N} \geq M$ is an upper constraint on the signal space dimension (for example, in a pulsed radar system $N$ is the number of coded pulses transmitted by each antenna and elaborated by the receiver, and $\overline{N}$ is the maximum number pulses that can be jointly processed);
- $\boldsymbol{A} \in \mathbb{C}^{N\times M}$ is the space-time code-matrix, whose $m$-th column represents the codeword transmitted by the $m$-th antenna;

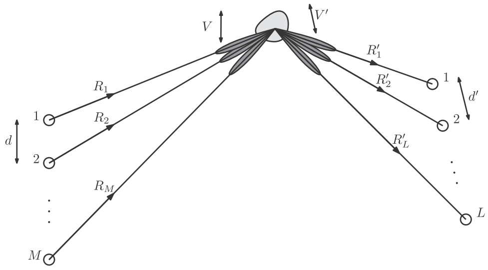


**Fig. 1.** Target scattering in an $M\times L$ radar.

- $\boldsymbol{\alpha}_\ell \in \mathbb{C}^M$ is the unknown random target response (the $m$-th entry is the scattering from antenna $m$ to antenna $\ell$);
- $\boldsymbol{w}_\ell \in \mathbb{C}^N$ is the overall disturbance (e.g., thermal noise and reverberation from the environment).

The observations at the $L$ receive nodes can be cast in the $LN$-dimensional vector

$$\boldsymbol{r} = (\boldsymbol{r}_1^T \cdots \boldsymbol{r}_L^T)^T = \begin{cases} (\boldsymbol{I}_L \otimes \boldsymbol{A})\boldsymbol{\alpha} + \boldsymbol{w} & \text{under } H_1 \\ \boldsymbol{w} & \text{under } H_0 \end{cases}$$

where $\boldsymbol{\alpha} = (\boldsymbol{\alpha}_1^T \cdots \boldsymbol{\alpha}_L^T)^T$ and $\boldsymbol{w} = (\boldsymbol{w}_1^T \cdots \boldsymbol{w}_L^T)^T$. We assume that $\boldsymbol{\alpha}$ and $\boldsymbol{w}$ are mutually independent, zero-mean, random vectors with covariance matrices $\boldsymbol{Q}_{\boldsymbol{\alpha}} \otimes \boldsymbol{R}_{\boldsymbol{\alpha}}$ and $\boldsymbol{Q}_{\boldsymbol{w}} \otimes \boldsymbol{R}_{\boldsymbol{w}}$, respectively, where $\boldsymbol{Q}_{\boldsymbol{\alpha}} \in \mathcal{M}_L$, $\boldsymbol{R}_{\boldsymbol{\alpha}} \in \mathcal{M}_M$, $\boldsymbol{Q}_{\boldsymbol{w}} \in \mathcal{M}_L$, and $\boldsymbol{R}_{\boldsymbol{w}} \in \mathcal{M}_N$, with $\boldsymbol{Q}_{\boldsymbol{w}}$ and $\boldsymbol{R}_{\boldsymbol{w}}$ full-rank. In this setting, $\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^H)$ represents the radiated energy, while the SDR under the hypothesis $H_1$ can be defined as [4,9]

$$\begin{aligned} \mathrm{SDR} &= \mathbb{E}[\|(\boldsymbol{Q}_{\boldsymbol{w}} \otimes \boldsymbol{R}_{\boldsymbol{w}})^{-1/2}(\boldsymbol{I}_L \otimes \boldsymbol{A})\boldsymbol{\alpha}\|^2] \\ &= \mathrm{tr}((\boldsymbol{Q}_{\boldsymbol{w}}^{-1/2}\boldsymbol{Q}_{\boldsymbol{\alpha}}\boldsymbol{Q}_{\boldsymbol{w}}^{-1/2}) \otimes (\boldsymbol{R}_{\boldsymbol{w}}^{-1/2}\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{\alpha}}\boldsymbol{A}^H\boldsymbol{R}_{\boldsymbol{w}}^{-1/2})) \\ &= \sum_{\ell=1}^{L}\sum_{i=1}^{\Delta} \lambda_\ell(\boldsymbol{Q}_{\boldsymbol{w}}^{-1/2}\boldsymbol{Q}_{\boldsymbol{\alpha}}\boldsymbol{Q}_{\boldsymbol{w}}^{-1/2})\lambda_i(\boldsymbol{R}_{\boldsymbol{w}}^{-1/2}\boldsymbol{A}\boldsymbol{R}_{\boldsymbol{\alpha}}\boldsymbol{A}^H\boldsymbol{R}_{\boldsymbol{w}}^{-1/2}) \end{aligned} \tag{2}$$

where $\Delta = \min\{M,N\}$.

### 2.1. Discussion of the scattering and disturbance model

As shown in [2], the degree of spatial correlation among the entries of $\boldsymbol{\alpha}$ depends upon the range and the extension of the target, on top of the carrier wavelength $\lambda$. To see this, consider the MIMO radar architecture in Fig. 1, where $V$ and $V'$ is the target extension in the transmit and receive sensor alignment direction, respectively, and $d$ and $d'$ is the sensor spacing at the transmitter and receiver. The scattering towards receive element $\ell$ can be modeled through a beam of angular width $\lambda/V'$, and the arc illuminated at distance $R'_\ell$ has length $\lambda R'_\ell/V'$: as a consequence, uncorrelated scattering towards different sensors occurs whenever the spacing $d'$ satisfies the condition $\lambda R'_\ell/V' < d'$, for $\ell=1,\dots,L$. Similarly at the transmitter side the uncorrelated scattering condition translates to $\lambda R_m/V < d$, $m=1,\dots,M$. This discussion suggests that the scattering model introduced in Section 2 is accurate in the following scenarios:

- Case 1: $\lambda R'_\ell/V' \gg d'\ \forall \ell$ and $\lambda R_m/V \gg d\ \forall m$.
  All target looks are fully correlated, whereby we can write $\boldsymbol{\alpha} = \theta(\boldsymbol{q} \otimes \boldsymbol{p})$ where $\theta$ is a zero-mean random variable with variance $\sigma^2$, $\boldsymbol{p}$ is the transmit steering