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Non-fragile distributed filtering for fuzzy systems with multiplicative gain variation

Dan Zhang [a,*], Peng Shi [b,c,d], Wen-An Zhang [a], Li Yu [a]

[a] Department of Automation, Zhejiang University of Technology, Hangzhou 310023, PR China
[b] School of Electrical and Electronic Engineering, University of Adelaide, Adelaide, SA 5005, Australia
[c] College of Automation, Harbin Engineering University, China
[d] College of Engineering and Science, Victoria University, Australia

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ABSTRACT

This paper is concerned with the non-fragile distributed H_∞ filtering problem for a class of discrete-time Takagi–Sugeno (T–S) systems with multiplicative gain variation. The multiplicative gain uncertainties reflecting the imprecision of the filter implementation are firstly described. Based on the fuzzy Lyapunov functional approach, a sufficient condition is presented such that the filtering error system is asymptotically stable with a prescribed H_∞ performance level. The filter gain parameters are determined by solving an optimization problem. Application on the robotic manipulator is given to show the effectiveness of the proposed new design method.

1. Introduction

It is well known that state estimation is an active research area with many practical applications in signal processing, communications and so on [1]. So far, many effective methodologies have been developed for the filter designs. One approach to this problem is H_∞ filtering, and the advantage of this approach is that the exact statistics on noise signals is not required [2], which is more robust than that of the Kalman filtering approach. On the other hand, fuzzy systems of the Takagi–Sugeno (T–S) model have attracted great interests from the control community. Much effort has been devoted onto the filtering of the T–S fuzzy systems, see [3–8] and the references therein. It is worth pointing out that all the above works are based on an implicit assumption that filters can be implemented exactly. However, inaccuracies or uncertainties do occur in filter implementation. Such uncertainties exist due to unexpected errors during the filter implementation, e.g., due round off errors in numerical computation and programming errors [9]. Thus, the filter should be designed such that it is insensitive to some amount of errors with respect to its gain, i.e., the designed filter is non-fragile. Basically, there are two types of gain variation, i.e., additive and multiplicative. Recently, the non-fragile filtering for fuzzy systems has been reported in [10–12]. Specifically, the authors in [12] have investigated the non-fragile filtering for continuous-time fuzzy systems with additive gain variation. A sufficient condition for the non-fragile H_∞ filter design is proposed in terms of linear matrix inequalities (LMIs). They showed that the explicit expression of desired H_∞ filter can be obtained when those LMIs are feasible. Nevertheless, the above non-fragile filters for fuzzy system are designed in the centralized way, and such a centralized design is fragile when some unexpected faults occur.

* Corresponding author.
E-mail addresses: danzhang@zjut.edu.cn (D. Zhang), peng.shi@adelaide.edu.au (P. Shi).

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With the development of the wireless sensor networks (WSNs) technique, the distributed filtering becomes possible [13]. In the distributed filtering system, each filter generates the estimation by using not only the local information but also the one from the neighbors. For example, the distributed finite-horizon filtering problem for a class of time-varying systems over lossy sensor networks was considered in [14], and the time-varying system (target plant) is subject to randomly varying nonlinearities (RVNs) caused by environmental circumstances. While the lossy sensor network suffers from quantization errors and successive packet dropouts that are described in a unified framework. A sufficient condition was established for the desired distributed finite-horizon filter to ensure that the prescribed average filtering performance constraint is satisfied. The solution of the distributed filter gains was characterized by solving a set of recursive linear matrix inequalities. Recent advances on the distributed filtering in sensor network can be found in [14–17] and the references therein. It should be pointing out that there has been little effort devoted on the non-fragile distributed filtering for dynamic systems, let alone the T–S fuzzy systems. Very recently, we have made a first attempt to the non-fragile distributed filtering for fuzzy system in [18], but only the additive filter gain variation problem was considered. To the best of the authors' knowledge, the non-fragile distributed filtering for fuzzy systems with multiplicative gain variation has not been studied yet, which motivates the present study. This work is challenging as the filter gain parameters are multiplying with the uncertainties, and such a complex coupling will bring much difficulty on the modeling and analysis of the filtering error system.

We focus on the non-fragile distributed filtering for discrete-time Takagi–Sugeno (T–S) systems in this paper, and the multiplicative gain variation problem is considered in the filter design. Based on the fuzzy Lyapunov function method, a sufficient condition is presented that guarantees the asymptotic stability of the filtering error system, and a descried H_∞ performance level is also ensured. The explicit filter gain parameters can be determined by solving an optimization problem. Finally, a case study on the one link robotic manipulator is proposed to show the effectiveness of the proposed new design method.

Notation: The notation used in this paper is standard. R^n denotes the n-dimensional Euclidean space. $l_2[0,\infty)$ is the space of square summable sequences. A^T represents the transpose of the matrix A. The symbol diag$\{A\}$ indicates that A is a diagonal matrix. row$\{A_{pi}\}$ represents a row matrix, that is row$\{A_{pi}\}=[A_{1i}\ \cdots\ A_{ni}]$, and row$\{A_{pij}\}=\left[A_{1ij}\ \cdots\ A_{nij}\right]$. col$\{A_{pij}\}$ represents a column matrix, that is col$\{A_{pij}\}=\left[A_{1ij}^T\ \cdots\ A_{nij}^T\right]^T$. The symbol $\otimes$ denotes the Kronecker product, and the symbol "$*$" is used in some matrix expressions to represent the symmetric terms. The dimension of a matrix is compatible as it is not explicitly defined, e.g., I stands for the identity matrix with appropriate dimension.

2. Problem formulation

Different from the centralized filtering system, there is no centralized estimation center in the distributed filtering system. Every sensor in the network acts also as an estimator, and they share each local measurement and estimation with the neighboring ones, see Fig. 1.

Standard definitions for the sensor networks are given as follows. Let the topology of a given sensor network be represented by a direct graph $\pi(k)=(\vartheta,\chi,\mathrm{A})$ of order n with the set of sensors $\vartheta=1,2,\ldots,n$, set of edges $\chi\subseteq\vartheta\times\vartheta$, and a weighted adjacency matrix $\mathrm{A}=[a_{pq}]$ with nonnegative adjacency elements a_{pq}. An edge of π is denoted by (p,q). The adjacency elements associated with the edges of the graph are $a_{pq}=1$, $(p,q)\in\vartheta$, if sensor p receives information from sensor q. Whereas $a_{pq}=0$, if sensor p cannot receive information from sensor q. Moreover, we assume $a_{pp}=1$ for all $p\in\vartheta$. The set of neighbors of node $p\in\vartheta$ plus the node itself are denoted by $N_p=q\in\vartheta:(p,q)\in\vartheta$. $\mathrm{A}=[a_{pq}]$ is a square matrix representing the topology of the sensor network.

In this paper, the target plant is described by the following T–S model:

Plant rule i: IF $\phi_1(k)$ is ψ_{i1} and $\phi_2(k)$ is $\psi_{i2}\ldots$ and $\phi_t(k)$ is ψ_{it}, THEN

$$\begin{cases} x(k+1)=A_ix(k)+B_iw(k), \\ z(k)=L_ix(k), \quad i=1,2,\ldots,r \end{cases} \tag{1}$$

where $\phi(k)=[\phi_1(k),\phi_2(k),\ldots,\phi_t(k)]$ is the premise variable vector, ψ_{ij} is the fuzzy set and r is the number of IF–THEN rules such that $i\in\Gamma=1,2,\ldots,r$. $x(k)\in\mathrm{R}^{n_1}$ is the state vector. $z(k)\in\mathrm{R}^{n_2}$ is the signal to be estimated. $w(k)\in\mathrm{R}^{n_3}$ is the disturbance signal, belonging to $l_2[0,\infty)$. A_i,B_i and L_i are known constant matrices with appropriate dimensions.

By using a center-average defuzzifier, product fuzzy inference, and a singleton fuzzifier, (1) can be rewritten as

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}h_i(\phi(k))[A_ix(k)+B_iw(k)], \\ z(k)=\sum_{i=1}^{r}h_i(\phi(k))[L_ix(k)], \end{cases} \tag{2}$$

where $h_i(\phi(k))=\omega_i(\phi(k))/\sum_{i=1}^{r}\omega_i(\phi(k))$ and $\omega_i(\phi(k))=\prod_{j=1}^{p}\psi_{ij}(\phi_j(k))$, with $\psi_{ij}(\phi_j(k))$ representing the grade of membership of $\phi_j(k)$ in ψ_{ij}. It is assumed that $\omega_i(\phi(k))\geq 0$ and $\sum_{i=1}^{r}\omega_i(\phi(k))>0$ for all $\phi(k)$.

The measurement model for the pth $(p=1,2,\ldots,n)$ sensor is given as

$$y_p(k)=C_{pi}x(k)+D_{pi}w(k), \tag{3}$$

where $y_p(k)\in\mathrm{R}^{n_4}$ is the measurement output measured by senor p from plant. The matrices C_{pi} and D_{pi} above are some known constant matrices with appropriate dimensions.

We now propose the following filter in the pth sensor: