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Variable step-size normalized LMS algorithm by approximating correlation matrix of estimation error

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ABSTRACT

In this letter, we propose a variable step-size normalized least mean square (NLMS) algorithm. We study the relationship among the NLMS, recursive least square and Kalman filter algorithms. Based on the relationship, we derive an equation to determine the step-size of NLMS algorithm at each time instant. In steady state, the convergence of the proposed algorithm is verified by using the equation, which describes the relationship among the mean-square error, excess mean-square error, and measurement noise variance. Through computer simulation results, we verify the performance of the proposed algorithm and the change in the variable step-size over iterations.

1. Introduction

The normalized least mean square (NLMS) algorithm is an adaptive filter algorithm that is simple and easy to implement. There have been several studies on improving its performance [1]. A variable step-size is one of the improvements suggested for the NLMS algorithm [2,3].

In this letter, we propose a variable step-size NLMS algorithm and the motivation of the proposed algorithm is the state-space approach for adaptive filter algorithms [4,5]. According to this approach, the adaptive filter algorithm can be derived by state-space equations; the NLMS algorithm is a special case of the state-space approach for adaptive filter algorithms. The relationship between the recursive least square (RLS) algorithm and the Kalman filter has been studied in [6]. We summarize the relationships and develop the proposed variable step-size NLMS algorithm by considering the relationship among the NLMS, RLS and Kalman filter algorithms.

Conventionally, most variable step-size algorithms are derived by minimizing a criterion or cost function to determine the step-size value [3,5]. In contrast, the proposed variable step-size algorithm is derived by approximating the correlation matrix of the estimation error. It does not require a differentiation operation to minimize the criterion for step-size. Moreover, the step-size calculation of the proposed algorithm is simple, and therefore, it does not pose a serious computational burden. The convergence of the proposed algorithm is confirmed by using the relationship that the summation of the excess mean-square error and variance of the measurement noise is equal to the mean-square error [7].

This letter is organized as follows. In Section 2, we summarize the relationship among the NLMS, RLS, and Kalman filter algorithms. In Section 3, we present the proposed variable step-size algorithm and verify the convergence. In Section 4, we show computer simulation results of the proposed algorithm and compare them to the variable step-size algorithm in [3,5] to verify the performance. In Section 5, we conclude this letter.

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2. Relationship among NLMS, RLS, and Kalman filter algorithms

A state-space equation without input force is given by

$$\mathbf{x}_{i+1} = \mathbf{A}_i\mathbf{x}_i + \mathbf{w}_i \tag{1}$$

$$\mathbf{y}_i = \mathbf{C}_i\mathbf{x}_i + \mathbf{v}_i \tag{2}$$

where $\mathbf{x}_i$, $\mathbf{y}_i$, $\mathbf{w}_i$ and $\mathbf{v}_i$ are the state, measurement, process noise, and measurement noise vectors, respectively, at time instant i [6]. We assume that all vectors are column vectors. The matrices $\mathbf{A}_i$ and $\mathbf{C}_i$ are the state transition and measurement matrices, respectively, at time instant i [6]. The Kalman filter is an algorithm to estimate the state vector when process and measurement noise exist. For $i=1,2,\ldots$, the Kalman filter equations can be written as follows [6]:

$$\mathbf{e}_i = \mathbf{y}_i - \mathbf{C}_i\hat{\mathbf{x}}_{i|i-1} \tag{3}$$

$$\mathbf{K}_i = \mathbf{P}_{i|i-1}\mathbf{C}_i^T[\mathbf{C}_i\mathbf{P}_{i|i-1}\mathbf{C}_i^T + \mathbf{S}_i]^{-1} \tag{4}$$

$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + \mathbf{K}_i\mathbf{e}_i \tag{5}$$

$$\hat{\mathbf{x}}_{i+1|i} = \mathbf{A}_i\hat{\mathbf{x}}_{i|i} \tag{6}$$

$$\mathbf{P}_i = [\mathbf{I} - \mathbf{K}_i\mathbf{C}_i]\mathbf{P}_{i|i-1} \tag{7}$$

and

$$\mathbf{P}_{i+1|i} = \mathbf{A}_i\mathbf{P}_i\mathbf{A}_i^T + \mathbf{Q}_i \tag{8}$$

where $\hat{\mathbf{x}}_{i|i-1}$ is an estimation of $\mathbf{x}_i$ based on the measurements $\mathbf{y}_1,\ldots,\mathbf{y}_{i-1}$. If we write the estimation error as $\tilde{\mathbf{x}}_i = \mathbf{x}_i - \hat{\mathbf{x}}_{i|i-1}$, $\mathbf{P}_{i|i-1}$ is the correlation matrix of the estimation error, i.e., $E[\tilde{\mathbf{x}}_i\tilde{\mathbf{x}}_i^T]$, where E is the expectation operator. $\mathbf{S}_i$ and $\mathbf{Q}_i$ are the correlation matrices of $\mathbf{v}_i$ and $\mathbf{w}_i$, respectively. Eqs. (3)–(8) are repeated with the given initial conditions $\mathbf{x}_{1|0}$ and $\mathbf{P}_{1|0}$, and the state vector is iteratively estimated [6].

Adaptive filter algorithms are used for channel estimation or noise cancelation [8]. Generally, they assume the channel to be estimated as a finite impulse response (FIR) model. If the tap-length of the impulse response sequence of the FIR model is given by M, the FIR model is defined as an $M\times 1$ vector $\mathbf{h}_i$, which is called the coefficient vector at time instant i. The measured signal for the adaptive filter algorithms is modeled as

$$d(i) = \mathbf{u}_i^T\mathbf{h}_i + \nu(i) \tag{9}$$

$$= y(i) + \nu(i) \tag{10}$$

where $\mathbf{u}_i$ is an input signal vector, $d(i)$ is a measured signal, and $\nu(i)$ is a measurement noise. The adaptive filter algorithms recursively estimate the coefficient vector.

In adaptive filter theory, there are two important algorithms: the NLMS and RLS algorithms [6]. The RLS algorithm can be regarded as a special case of the Kalman filter. If we write the state-space equation with its state vector as the coefficient vector, that is,

$$\mathbf{h}_{i+1} = \mathbf{h}_i \tag{11}$$

$$d(i) = \mathbf{u}_i^T\mathbf{h}_i + \nu(i) \tag{12}$$

the corresponding Kalman filter equations can be written as

$$e(i) = d(i) - \mathbf{u}_i^T\hat{\mathbf{h}}_{i|i-1} \tag{13}$$

$$\mathbf{K}_i = \mathbf{P}_{i|i-1}\mathbf{u}_i[\mathbf{u}_i^T\mathbf{P}_{i|i-1}\mathbf{u}_i + \sigma_\nu^2]^{-1} \tag{14}$$

$$\hat{\mathbf{h}}_{i|i} = \hat{\mathbf{h}}_{i|i-1} + \mathbf{K}_i e(i) \tag{15}$$

$$\hat{\mathbf{h}}_{i+1|i} = \hat{\mathbf{h}}_{i|i} \tag{16}$$

$$\mathbf{P}_i = [\mathbf{I} - \mathbf{K}_i\mathbf{u}_i^T]\mathbf{P}_{i|i-1} \tag{17}$$

and

$$\mathbf{P}_{i+1|i} = \mathbf{P}_i \tag{18}$$

Eqs. (16) and (18) are trivial, so we can rewrite the equations as

$$e(i) = d(i) - \mathbf{u}_i^T\hat{\mathbf{h}}_i \tag{19}$$

$$\mathbf{K}_i = \mathbf{P}_i\mathbf{u}_i[\mathbf{u}_i^T\mathbf{P}_i\mathbf{u}_i + 1]^{-1} \tag{20}$$

$$\hat{\mathbf{h}}_{i+1} = \hat{\mathbf{h}}_i + \mathbf{K}_i e(i) \tag{21}$$

and

$$\mathbf{P}_{i+1} = \mathbf{P}_i - \mathbf{K}_i\mathbf{u}_i^T\mathbf{P}_i \tag{22}$$

For the above equations, we simplify the subscripts such that $\hat{\mathbf{h}}_{i|i-1} = \hat{\mathbf{h}}_i$ and $\mathbf{P}_{i|i-1} = \mathbf{P}_i$; we assume that σ_ν^2 is unity. These equations are the same as the update equations of the standard RLS algorithms [6]. Therefore, we conclude that the Kalman filter applied to the state-space equations (11) and (12) are equal to the RLS algorithm when the power of the measurement noise is assumed to be unity.

If we set $\mathbf{P}_i=\mathbf{I}$ for all time instants i and ignore updating $\mathbf{P}_i$ in (22), the equations of the RLS algorithm become

$$e(i) = d(i) - \mathbf{u}_i^T\hat{\mathbf{h}}_i \tag{23}$$

$$\mathbf{K}_i = \mathbf{u}_i[\mathbf{u}_i^T\mathbf{u}_i + 1]^{-1} \tag{24}$$

and

$$\hat{\mathbf{h}}_{i+1} = \hat{\mathbf{h}}_i + \mathbf{K}_i e(i) \tag{25}$$

These equations are the same as the standard NLMS algorithm (26) when the step size μ and regularization factor ε are both unity:

$$\hat{\mathbf{h}}_{i+1} = \hat{\mathbf{h}}_i + \mu\mathbf{u}_i[\mathbf{u}_i^T\mathbf{u}_i + \varepsilon]^{-1}(d(i) - \mathbf{u}_i^T\hat{\mathbf{h}}_i) \tag{26}$$

Based on these considerations, we can determine the relationship among the Kalman filter, RLS and NLMS algorithms. In the next section, we propose a variable step-size algorithm for the NLMS algorithm by using the relationship.

3. Proposed algorithm

3.1. Derivation of proposed algorithm

As stated in the previous section, the NLMS algorithm assumes $\mathbf{P}_i=\mathbf{I}$ for all time instants i. This assumption is a rough approximation of $\mathbf{P}_i$. For a better approximation of $\mathbf{P}_i$, we set $\mathbf{P}_i = \mathrm{diag}(\lambda_i)$, where $\mathrm{diag}(\lambda_i)$ is a diagonal matrix