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H_∞ filtering for time-delayed singular Markovian jump systems with time-varying switching: A quantized method

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ABSTRACT

This paper discusses the H_∞ filtering problem of time-delayed singular Markovian jump systems (SMJSs) with time-varying transition rate matrix (TRM). In this paper, the underlying TRM is studied by the quantized method, whose difficulties used in system analysis and synthesis with infinite precision are overcome. Based on the proposed quantization principle, the time-varying TRM is firstly quantized into a series of finite TRMs with norm bounded uncertainties. Then, new criteria depending on time delay and quantization density simultaneously are developed such that the corresponding system is stochastic admissible with an H_∞ performance. Several sufficient conditions for the existence of the desired filter are given in the form of linear matrix inequalities (LMIs). Finally, numerical examples are used to demonstrate the correctness and superiorities of the proposed methods.

1. Introduction

In the past decades, the filtering problem has been studied as a hot topic. It is well-known that the filtering technique is very useful to estimate unavailable states of a given system through noisy measurement, which plays important roles in the areas of control and signal processing. When the system and the statistics of noise disturbances are known exactly, Kalman filtering approach [1–4] is effective. But sometimes, such assumptions are difficult to be satisfied. In order to remove these limitations, another filtering technique referred to H_∞ filtering has been proposed. Compared with Kalman filtering technique, H_∞ filtering has powerful signal estimation and good robustness performance. Such features have motivated the study of H_∞ filtering problem for variant systems, see [5–10] and the references therein.

Markovian jump system (MJS) such as [11,12] is usually modeled by a class of subsystems driven by a Markov chain taking values in a finite set. Two kinds of mechanisms are simultaneously contained in such a system, which are named as time-evolving and event-driven mechanisms respectively. The first one is continuous in time and related to the sate vector; the latter one is referred to be operation mode or system mode and driven by a Markov process. Many practical systems experiencing system parameters or structures changed abruptly, such as aircraft control, manufacturing system, power systems, and solar receiver system, are very appropriately described by MJSs. Up to now, the H_∞ filtering problems of various kinds of MJSs have emerged, see, e.g., [13–18]. Especially, singular Markovian jump systems (SMJSs) [19–22] are usually used to describe singular systems having abrupt changes. Due to singular systems having additional impulsive and non-dynamic modes, the analysis and synthesis of such systems are usually more complicated than normal systems. In singular systems, the regularity, impulse elimination

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(for continuous case) or causality (for discrete case) and stability, should be considered simultaneously. During the past decades, many research topics on such systems without or with time delays have emerged in [23–27]. When there are time delays in SMJSs, the H_∞ filtering problem was considered in [28–30].

As we know, when an MJS is considered, the underlying TRM plays important roles in system analysis and synthesis. By investigating the above references, it is seen that most results of MJSs including SMJSs have an assumption that TRM should be available exactly. This is not true in many practical applications. Instead, the corresponding TRM may be more general such as uncertain or partially unknown. When an TRM has uncertainties, some results for normal MJSs were presented in [31,32], while similar problems of SMJSs were considered in [26,30]. When an TRM is partially unknown that some elements are unknown, some problems for MJSs including SMJSs were reported in [33–36]. Though many problems of MJSs with general TRMs referred above were studied, there are very few references to discuss another general case of time-varying TRM. Due to TRM time-varying, it is impossible to use it with infinite precision in system analysis and synthesis. Thus, how to deal with the time-varying TRM appropriately becomes the first problem to be solved. To the best of our knowledge, up to now, there is no reference to report this issue for MJSs with time-varying TRM. In this paper, we will apply the quantized method to handle the time-varying TRM. That is because the quantization is widely applied to digital control systems. It is seen that the utility of the quantizer is to convert the continuous signal into piecewise continuous signals. Recent years, more and more attention is focused on the quantization problem, and several kinds of quantization such as state quantization, input quantization, output quantization and measured output quantization, have been presented in [37–41]. In these references, the proposed quantizers are applied to a vector, and there are on constraints on quantization tactics. In this paper, because the developed quantized method will be applied to the time-varying TRM, there will be some quantization constraints on the proposed quantizer. When the TRM is time-varying, the related problems of SMJSs have not been fully investigated and still remains.

In this paper, we focus on the H_∞ filtering for a class of time-delayed SMJSs with time-varying TRM, where the time-varying TRM is handled by a quantized method. Based on the developed quantization principle, the time-varying TRM is quantized into finite TRMs having norm bounded uncertainties. Thereby, the difficulties of time-varying TRM used in system analysis and synthesis are overcome. New delay-dependent results on stochastic admissibility with an H_∞ index are established, which also depend on the quantization density. By the LMI approach, several conditions for the solvability of an H_∞ filter are proposed. Finally, simulation results are used to show the effectiveness and advantage of the proposed methods.

Notation: $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\mathcal{E}(\cdot)$ is the expectation operator with respect to some probability measure. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, $diag\{\cdots\}$ for a block-diagonal matrix, and $M^\star \triangleq M+M^T$.

2. Preliminaries

Considering a class of time-delayed SMJSs with time-varying TRM described as

$$\begin{cases} E\dot{x}(t)=A(r_t)x(t)+A_d(r_t)x(t-\tau)+B(r_t)\omega(t) \\ y(t)=C(r_t)x(t)+C_d(r_t)x(t-\tau)+D(r_t)\omega(t) \\ z(t)=L(r_t)x(t) \\ x(t)=\phi(t), \quad \forall t\in[-\tau,0] \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $\omega(t)\in\mathbb{R}^m$ belonging to $\mathcal{L}_2[0,\infty)$ is the disturbance input, $y(t)\in\mathbb{R}^p$ and $z(t)\in\mathbb{R}^q$ are measurement and signal to be estimated respectively. Matrix $E\in\mathbb{R}^{n\times n}$ may be singular with $rank(E)=r\le n$. Delay τ is an unknown constant and satisfies $0\le\tau\le\overline{\tau}$, where $\overline{\tau}$ is constant and satisfies $\overline{\tau}\ge 0$. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r_t)$ and $L(r_t)$ are known matrices of compatible dimensions. The operation mode $\{r_t, t\ge 0\}$ is assumed to be a continuous-time Markov process with time-varying TRM $\Pi(t)=(\pi_{ij}(t))\in\mathbb{R}^{N\times N}$ given by

$$Pr\{r_{t+h}=j|r_t=i\}=\begin{cases} \pi_{ij}(t)h+o(h), & j\ne i \\ 1+\pi_{ij}(t)h+o(h), & j=i \end{cases} \tag{2}$$

where $h>0$, $\lim_{h\to 0^+}(o(h)/h)=0$, and $\pi_{ij}(t)\ge 0$, $\forall t\ge 0$, if $i\ne j$, $\pi_{ii}(t)=-\sum_{j\ne i}\pi_{ij}(t)$.

In this paper, we will design the following filter to estimate $z(t)$, though the TRM is time-varying. The desired filter is described as

$$\begin{cases} E\dot{x}_f(t)=A_f(r_t)x_f(t)+B_f(r_t)y(t) \\ z_f(t)=L_f(r_t)x_f(t) \end{cases} \tag{3}$$

where $x_f(t)\in\mathbb{R}^{n\times n}$, $A_f(r_t)$, $B_f(r_t)$ and $L_f(r_t)$ are filter parameters to be determined. Combining filter (3) with system (1), when $r_t=i\in\mathbb{S}$, we have the filtering error system:

$$\begin{cases} \overline{E}\dot{\overline{x}}(t)=\overline{A}_i\overline{x}(t)+\overline{A}_{di}\overline{x}(t-\tau)+\overline{B}_i\omega(t) \\ \overline{z}(t)=\overline{L}_i\overline{x}(t) \end{cases} \tag{4}$$

where

$$\overline{E}=\begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \quad \overline{x}(t)=\begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \quad \overline{A}_i=\begin{bmatrix} A_i & 0 \\ B_{fi}C_i & A_{fi} \end{bmatrix}$$

$$\overline{A}_{di}=\begin{bmatrix} A_{di} & 0 \\ B_{fi}C_{di} & 0 \end{bmatrix}, \quad \overline{B}_i=\begin{bmatrix} B_i \\ B_{fi}D_i \end{bmatrix}$$

$$\overline{z}(t)=z(t)-z_f(t), \quad \overline{L}_i=[L_i \ \ -L_{fi}]$$

Similar to [19,20], some definitions for system (4) are needed throughout this paper.

Definition 1. Time-delayed SMJS (4) with time-varying TRM (2) is said to be

(1) regular and impulse-free for any constant time delay τ satisfying $0\le\tau\le\overline{\tau}$, if the pair $(\overline{E},\overline{A}_i)$ is regular and impulse-free for any $i\in\mathbb{S}$;
(2) stochastically stable, if there exists a constant $M(\phi(t),r_0)$ such that

$$\mathcal{E}\left(\int_0^\infty x^T(t)x(t)\,dt|\phi(t),r_0\right)\le M(\phi(t),r_0)$$

for any initial conditions $\phi(t)\in\mathbb{R}^n$ and $r_0\in\mathbb{S}$;