Signal Processing 109 (2015) 84–94

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Legendre nonlinear filters ☆

Alberto Carini [a,*], Stefania Cecchi [b], Laura Romoli [b], Giovanni L. Sicuranza [c]

[a] DiSBeF, University of Urbino Carlo Bo, Urbino, Italy
[b] DII, Università Politecnica delle Marche, Ancona, Italy
[c] DIA, University of Trieste, Trieste, Italy

A R T I C L E I N F O

Article history:
Received 8 July 2014
Received in revised form
2 October 2014
Accepted 31 October 2014
Available online 11 November 2014

Keywords:
Nonlinear filters
Nonlinear system identification
Linear-in-the-parameters nonlinear filters
Legendre nonlinear filters
Perfect periodic sequences
Cross-correlation method

A B S T R A C T

The paper discusses a novel sub-class of linear-in-the-parameters nonlinear filters, the Legendre nonlinear filters. The novel sub-class combines the best characteristics of truncated Volterra filters and of the recently introduced even mirror Fourier nonlinear filters, in particular: (i) Legendre nonlinear filters can arbitrarily well approximate any causal, time-invariant, finite-memory, continuous, nonlinear system; (ii) their basis functions are polynomials, specifically, products of Legendre polynomial expansions of the input signal samples; (iii) the basis functions are also mutually orthogonal for white uniform input signals and thus, in adaptive applications, gradient descent algorithms with fast convergence speed can be devised; (iv) perfect periodic sequences can be developed for the identification of Legendre nonlinear filters. A periodic sequence is perfect for a certain nonlinear filter if all cross-correlations between two different basis functions, estimated over a period, are zero. Using perfect periodic sequences as input signals permits the identification of the most relevant basis functions of an unknown nonlinear system by means of the cross-correlation method. Experimental results involving identification of real nonlinear systems illustrate the effectiveness and efficiency of this approach and the potentialities of Legendre nonlinear filters.

1. Introduction

The paper discusses a novel sub-class of finite memory linear-in-the-parameters (LIP) nonlinear filters. LIP nonlinear filters constitute a very broad filter class, which includes most of the commonly used finite-memory and infinite-memory nonlinear filters. The class is characterized by the property that the filter output depends linearly on the filter coefficients. It includes the well known truncated Volterra filters [1], which are still actively studied and used in applications [2–9], but also other popular polynomials filters, as the Hammerstein filters [1,10–13], the memory and generalized memory polynomial filters [14,15], and non-polynomial filters based on functional expansions of the input samples, as the functional link artificial neural networks (FLANN) [16] and the radial basis function networks [17]. The interested reader can refer to [18] for a review under a unified framework of finite-memory LIP nonlinear filters. Infinite-memory LIP nonlinear filters have also been studied [19–24] and used in applications.

Recently, the finite memory LIP class has been enriched with two novel sub-classes: the Fourier Nonlinear (FN) filters [25,26] and the Even Mirror Fourier Nonlinear (EMFN) filters [26,27]. FN and EMFN filters can be originated from the truncation of a multidimensional Fourier

☆ This work has been supported in part by DiSBeF Research Grant (no. 84/2013-DiSBeF-24_5). Parts of this paper have been published in the Proceedings of ICASSP 2014 (A. Carini, S. Cecchi, M. Gasparini, G.L. Sicuranza, Introducing Legendre nonlinear filters, Florence, Italy, 2014), and in Proceedings of EUSIPCO 2014 (A. Carini, S. Cecchi, L. Romoli, G.L. Sicuranza, Perfect periodic sequences for Legendre nonlinear filters, Lisbon, Portugal, 2014).

* Corresponding author. Tel.: +39 0722 304412; fax: +39 0722 4475.
E-mail addresses: alberto.carini@uniurb.it (A. Carini), s.cecchi@univpm.it (S. Cecchi), l.romoli@univpm.it (L. Romoli), sicuranza@univ.trieste.it (G.L. Sicuranza).

http://dx.doi.org/10.1016/j.sigpro.2014.10.037
0165-1684/

series expansion of a periodic repetition or an even mirror periodic repetition, respectively, of the nonlinear function they approximate. FN and EMFN filters are based on trigonometric function expansions, as the FLANN filters, but in contrast to the latter, their basis functions form an algebra that satisfies all the requirements of the Stone–Weierstrass approximation theorem [28]. Consequently, they can arbitrarily well approximate any causal, time-invariant, finite-memory, continuous, nonlinear system. EMFN filters provide a much more compact representation of nonlinear systems than FN filters [26], and thus should be the preferred choice. It has been shown that EMFN filters can also be better models than Volterra filters in the presence of strong nonlinearities, while Volterra filters provide better results for weak or medium nonlinearities [26]. An interesting property of EMFN (and FN) filters, which is not shared by Volterra filters, derives from orthogonality of the basis functions for white uniform input signals in the range $[-1,+1]$. This property is particularly appealing since it allows the derivation of gradient descent algorithms with fast convergence speed and of efficient identification algorithms. In [29,30], it was shown that perfect periodic sequences (PPSs) can be developed for the identification of EMFN filters. PPSs have been extensively studied and proposed as inputs for linear system identification [31] and in this context they have found application in signal processing [32], information theory [33], communications [34,35], and acoustics [36]. A periodic sequence is called perfect for a modeling filter if all cross-correlations between two of its basis functions, estimated over a period, are zero. By applying as input signal a PPS, it is possible to model an unknown system exploiting the cross-correlation method, i.e., computing the cross-correlation between the basis functions and the system output. The most relevant basis functions, i.e., those that guarantee the most compact representation of the nonlinear system according to some information criterion, can also be easily estimated.

The novel sub-class of finite memory LIP nonlinear filters discussed in this paper is that of Legendre nonlinear (LN) filters, first introduced in [37]. LN filters combine the best characteristics of truncated Volterra and EMFN filters, as detailed in the following. First of all, the basis functions of LN filters are polynomials, as for Volterra filters. More specifically, they are products of Legendre polynomial expansions of the input samples that satisfy all the requirements of the Stone–Weierstrass approximation theorem. Therefore, LN filters are universal approximators, as well as Volterra, FN, and EMFN filters. With the term “universal approximators” we mean that these filters can arbitrarily well approximate any causal, time-invariant, finite-memory, continuous, nonlinear system. Secondly, the basis functions of LN filters are orthogonal for white uniform input signals in $[-1,+1]$, which means that they share all the benefits offered by FN and EMFN filters in terms of convergence speed of gradient descent adaptation algorithms and efficient identification algorithms. As a matter of fact, it is shown in Section 5 that the 2-norm condition number of the autocorrelation matrix of the input data vector for the Volterra filter is always larger than that of the EMFN and LN filters. As a consequence, EMFN and LN filters always provide a better convergence speed than a Volterra filter for white uniform input signals. Finally, as it was first shown in [38], PPSs can also be developed and used for the identification of LN filters. Indeed, they easily allow an efficient estimation of the most compact representation of the unknown nonlinear system, by using the cross-correlation approach and some information criterion. All these advantages come at the expense of a very small increase of the implementation complexity with respect to Volterra filters. All these aspects are considered in detail in the paper.

It is worth noting that LN filters are based on polynomial basis functions including the linear function, and thus their modeling capabilities are similar to those of Volterra filters. Therefore, LN filters can provide more compact models than EMFN filters for weak or medium nonlinearities. Moreover, identifying LN filters using PPSs is one of the most efficient methods for the identification of Volterra filters. Indeed, once the LN filter has been identified, it can be easily transformed into a Volterra filter representation exploiting the properties of Legendre polynomials.

The approach used in this paper to introduce the LN filter class can be applied to any family of orthogonal polynomials defined on a finite interval. Legendre polynomials are specifically considered since they have been already used for nonlinear filtering. Indeed, they have found application in Hammerstein models [39,40], FLANN filters [41–43], and neural networks [44]. Nevertheless, it should be noticed that the approaches of the literature do not make use of cross-terms, i.e., products among basis functions involving samples with different time delay, which can be very important for modeling nonlinear systems [18]. The corresponding basis functions do not form an algebra, because they are not complete under product. Thus, in contrast to the filters proposed in this paper, those previously considered are not universal approximators for causal, time-invariant, finite-memory, continuous, nonlinear systems.

Compared with the early conference contributions [37,38], in this paper we present an organic and detailed introduction of LN filters and their properties, discussing with particular attention PPSs for LN filters. Differently from [37], LN filters are introduced in this paper starting from a normalized set of Legendre polynomials. In contrast to [38], full proofs of properties of the PPSs for LN filters are here presented. Moreover, a discussion about the advantages and disadvantages of using LN filters and PPSs for system identification is also included in this paper.

The rest of the paper is organized as follows. Section 2 reviews basic concepts about LIP nonlinear filters, the Stone–Weierstrass theorem, and Legendre polynomials. Section 3 derives the LN filters and discusses their properties. Section 4 discusses PPSs for LN filters and their use for system identification. Section 5 presents experimental results that illustrate the advantages of LN filters and PPSs. Concluding remarks follow in Section 6.

Throughout the paper the mathematical notation of Table 1 is used. Moreover, sets are represented with curly brackets, intervals with square brackets, while the following convention for brackets $\{[(\cdots\{[()]\}\cdots)]\}$ is used elsewhere.