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Dissipativity analysis for fixed-point interfered digital filters

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A B S T R A C T

This paper is concerned with establishing a new criterion for the (Q, S, R)-α-dissipativity of fixed-point interfered state-space digital filters with saturation overflow arithmetic. The objective of this paper is to present the H_∞ performance, passivity, and mixed H_∞/passivity criteria in a unified framework. By tuning the weight matrices, the proposed criterion reduces to the H_∞ performance, passivity, and mixed H_∞/passivity criteria. Improved criteria are also proposed for reducing the conservatism of the proposed criterion. These criteria are expressed with linear matrix inequalities (LMIs). A numerical example shows the effectiveness of the proposed results.

1. Introduction

The digital filter is a very important building block in many engineering areas, such as electrical and electronic engineering. The implementation of a digital filter with fixed-point arithmetic in digital hardware is associated with nonlinearities, such as quantization and saturation, because of finite word length effects and a constraint imposed on the maximum bound of signals. These nonlinearities can generate undesired effects, such as zero-input limit cycles, in digital filters. Hence, the analysis and design of digital filters under these nonlinearities are extremely important. So far, much research has focused on the analysis of the stability of digital filters under nonlinearities [1–11].

The hardware implementation of a large-scale digital filter usually requires its division into several small-scale digital filters. In this situation, interferences between these small-scale filters always exist, resulting in poor performance or final destruction [12,13]. Thus, analysis of the effects of interferences in digital filters is an important research subject. Recently, Ahn tackled this issue and established some new stability criteria for one-dimensional and two-dimensional digital filters in [14–18] and [18–23], respectively.

The dissipativity concept [24,25], which originated from electrical networks, gives an important framework for synthesis and analysis of several control and signal processing systems using input–output descriptions with energy-based considerations. The input–output description leads to a modular approach to the synthesis and analysis of signal processing systems (for example, digital filters). One of the important properties of dissipative systems is that the total energy stored in the dissipative system decreases through time. It turns out that the dissipativity concept is a very helpful guide for the design of state estimation filters and output feedback controllers [26]. Dissipativity is regarded as a generalization of some well-known performance indices, such as H_∞ performance, passivity, and mixed H_∞/passivity. Thus, dissipativity provides a unified framework to cover H_∞ performance, passivity, and mixed H_∞/passivity [27–31]. For this reason, many researchers have used the dissipativity approach to create new controller and observer design methods for several nonlinear systems [26,32–35]. Here, an interesting question arises: Is it possible to obtain a dissipativity criterion for fixed-point state-space interfered digital filters? This paper answers this question in the positive. To the best of the authors' knowledge, the current

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literature contains no papers on the dissipativity of fixed-point state-space interfered digital filters using saturation overflow arithmetic.

This paper establishes a new (Q, S, R)-α-dissipativity criterion for fixed-point state-space interfered digital filters with saturation overflow arithmetic. The purpose of this paper is to provide a unified filter stability analysis approach for fixed-point digital filters with interferences. The proposed criterion covers the H_∞ performance, passivity, and mixed H_∞/passivity criteria as special cases by tuning the weight matrices. With the introduction of slack matrices and diagonally dominant matrices, improved (Q, S, R)-α-dissipativity criteria for fixed-point state-space digital filters are also proposed to reduce the conservatism of the proposed criterion. These criteria are described with linear matrix inequalities (LMIs) [36,37] and thus are computationally attractive.

This paper is organized as follows. In Section 2, we present an LMI-based criterion for the (Q, S, R)-α-dissipativity of fixed-point state-space interfered digital filters. In Section 3, improved (Q, S, R)-α-dissipativity criteria are proposed using slack matrices and diagonally dominant matrices. In Section 4, we investigate some special cases of the proposed criterion. In Section 5, a numerical example is given, and finally, conclusions are presented in Section 6.

2. (Q, S, R)-α-dissipativity criterion for fixed-point digital filters

Consider the following form of digital filter:

$$x(r+1)=f(y(r))+w(r)$$
$$=[f_1(y_1(r))f_2(y_2(r))\cdots f_n(y_n(r))]^T$$
$$+[w_1(r)w_2(r)\cdots w_n(r)]^T, \tag{1}$$

$$y(r)=Ax(r)+w(r), \tag{2}$$

where $x(r)=[x_1(r)x_2(r)\cdots x_n(r)]^T\in R^n$ is the state vector, $y(r)=[y_1(r)y_2(r)\cdots y_n(r)]^T\in R^n$ is the output vector, $w(r)=[w_1(r)w_2(r)\cdots w_n(r)]^T\in R^n$ is the external interference, and $A\in R^{n\times n}$ is the coefficient matrix. Here, we consider the following saturation nonlinearities:

$$f_i(y_i(r))=\begin{cases}1 & \text{if } y_i(r)>1,\\ y_i(r) & \text{if } -1\le y_i(r)\le 1,\quad i=1,2,\dots,n,\\ -1 & \text{if } y_i(r)<-1,\end{cases} \tag{3}$$

which are confined to the sector [0, 1], i.e.,

$$f_i(0)=0, 0\le\frac{f_i(y_i(r))}{y_i(r)}\le 1,\quad i=1,2,\dots,n. \tag{4}$$

In this paper, given a constant $\alpha\ge 0$ and constant matrices $Q\in R^{n\times n}$, $S\in R^{n\times n}$, and $R\in R^{n\times n}$ with Q and R symmetric, we obtain a new criterion such that the digital filter (1) and (2) satisfies

$$\sum_{r=0}^{T} y^T(r)Qy(r)+2\sum_{r=0}^{T} y^T(r)Sw(r)+\sum_{r=0}^{T} w^T(r)Rw(r)$$
$$\ge\alpha\sum_{r=0}^{T} w^T(r)w(r) \tag{5}$$

under the zero initial condition, where $T>0$. The digital filter is said to be (Q, S, R)-α-dissipative with the performance bound α if condition (5) is satisfied.

The following theorem gives a new (Q, S, R)-α-dissipativity criterion for state-space fixed-point digital filters.

Theorem 1. *Given a constant $\alpha\ge 0$ and constant matrices $Q\in R^{n\times n}$, $S\in R^{n\times n}$, and $R\in R^{n\times n}$ with Q and R symmetric, assume that there exist a symmetric positive definite matrix $P\in R^{n\times n}$, a positive definite diagonal matrix $M\in R^{n\times n}$, and a positive scalar δ such that $\Phi<0$, where*

$$\Phi=\begin{bmatrix}\delta A^TA-A^TQA-P & \star & \star\\ MA & P-\delta I-2M & \star\\ \delta A-S^TA-QA & P+M & P+(\delta+\alpha)I-R-Q-S-S^T\end{bmatrix} \tag{6}$$

and $\star$ denotes an entry that can be deduced from the symmetry of the matrix. Then, the digital filter (1) and (2) *is (Q, S, R)-α-dissipative with the performance bound α.*

Proof. Condition (4) implies

$$f^T(y(r))f(y(r))=f^T(Ax(r)+w(r))f(Ax(r)+w(r))$$
$$\le[Ax(r)+w(r)]^T[Ax(r)+w(r)]$$
$$=x^T(r)A^TAx(r)+w^T(r)Ax(r)+x^T(r)A^Tw(r)$$
$$+w^T(r)w(r), \tag{7}$$

from which we have

$$\delta[x^T(r)A^TAx(r)+w^T(r)Ax(r)+x^T(r)A^Tw(r)+w^T(r)w(r)$$
$$-f^T(y(r))f(y(r))]\ge 0. \tag{8}$$

Consider the following function: $V(x(r))=x^T(r)Px(r)$. Using (8), its time difference $(\Delta V(x(r))\triangleq V(x(r+1))-V(x(r)))$ satisfies

$$\Delta V(x(r))\le f^T(y(r))Pf(y(r))+f^T(y(r))Pw(r)+w^T(r)Pf(y(r))$$
$$+w^T(r)Pw(r)-x^T(r)Px(r)+2f^T(y(r))M[Ax(r)$$
$$+w(r)-f(y(r))]-2f^T(y(r))M[y(r)-f(y(r))]$$
$$+\delta[x^T(r)A^TAx(r)+w^T(r)Ax(r)+x^T(r)A^Tw(r)$$
$$+w^T(r)w(r)-f^T(y(r))f(y(r))]$$
$$=\Omega^T(r)\Phi\Omega(r)+y^T(r)Qy(r)+2y^T(r)Sw(r)$$
$$+w^T(r)[R-\alpha I]w(r)-2f^T(y(r))M$$
$$\times[y(r)-f(y(r))], \tag{9}$$

where

$$\Omega(r)=\begin{bmatrix}x(r)\\ f(y(r))\\ w(r)\end{bmatrix} \tag{10}$$

and the term $-2f^T(y(r))M[y(r)-f(y(r))]$ is not positive considering (3). Thus, if the LMI (6) is satisfied, we have

$$\Delta V(x(r))<y^T(r)Qy(r)+2y^T(r)Sw(r)+w^T(r)[R-\alpha I]w(r). \tag{11}$$

Summation of both sides of (11) from 0 to $T-1$ gives

$$\sum_{r=0}^{T} y^T(r)Qy(r)+2\sum_{r=0}^{T} y^T(r)Sw(r)+\sum_{r=0}^{T} w^T(r)[R-\alpha I]w(r)$$
$$>\sum_{r=0}^{T}\Delta V(x(r))=V(x(T+1))-V(x(0))\ge 0 \tag{12}$$

under the zero initial condition, which guarantees (5). This completes the proof. □