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Prior-exploiting Direction-of-Arrival algorithms for partially uncorrelated source signals ☆

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ABSTRACT

In this article, we investigate the performance of the recently proposed Direction-Of-Arrival (DOA) estimator POWDER (Prior Orthogonally Weighted Direction EstimatoR). The method is exploiting a specific form of prior information, namely that some DOAs are known, as well as that the correlation state between some of the source signals is known. In such scenarios, it is desirable to exploit the prior information already in the estimator design such that the knowledge can benefit the estimation of the DOAs of the unknown sources.

Through an asymptotical statistical analysis, we find closed form expressions for the accuracy of the method. We also derive the relevant Cramér–Rao Bound, and we show the algorithm to be efficient under mild assumptions. The realizable performance in the finite sample-case is studied through numerical Monte-Carlo simulations, from which one can conclude that the theoretically predicted accuracies are attained for modest sample sizes and comparatively low SNR. This has the implication that the algorithm is significantly more accurate than other, state-of-art, methods, in a wide range of scenarios.

1. Introduction

Direction-of-Arrival (DOA) estimation is a classical topic in signal processing and much work has been done in the area in the past decades. A passive array of sensors is receiving signals from a number of distinct sources, and the objective is to estimate the directions these signal are impinging from. In the seminal works [1] and [2], statistically efficient DOA estimation methods are presented. The underlying assumptions for the mentioned methods to be efficient were quite mild, notably, i.i.d. spatially white sensor noise. Thus, the mild assumptions give estimators that are applicable to a wide range of scenarios.

However, many different scenarios exist in practice, and in some of these more restrictive assumptions can be made; for example, some of the source directions might be known *a-priori*. Exploiting such information in the design of the estimator can be expected to produce methods that are more accurate than [1] and [2], and that this is indeed possible has been shown numerous times, e.g. [3–5]. Another example of a more restrictive assumption is that it might be known that the source signals are spatially uncorrelated, e.g. [6,7], and the combination of these two types of prior information was in [8] shown to be beneficial.

Recently, a new DOA algorithm denoted Prior Orthogonally Weighted Direction EstimatoR (POWDER), was proposed [9], where it was assumed that the known and the unknown DOAs are uncorrelated, but no assumptions were made on the

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correlation between the signals in the sets of known and unknown signals, respectively. Thus, the method of [9] is applicable in a wider set of scenarios than the one in [8], and it was also shown that for scenarios where both methods are app;licable, the former possess better small-sample performance than the latter. Additionally, in other scenarios, POWDER significantly outperformed state-of-the-art methods [1,5]. One example of a scenario where a corresponding data model is applicable is in wireless communications, where a transmitter at a known location is transmitting a signal which is uncorrelated to the signals emanating from the emitters at the unknown locations. The known location could correspond to, e.g., an interfering base station or wireless access point.

In this article we extend the work in [9] in the following ways: we derive closed form expressions for the asymptotic variances of POWDER; we derive the Cramér–Rao Bound (CRB) under the particular assumptions studied; we show that under benign conditions the POWDER-method attain the CRB; we conclude the article by, through numerical simulations, investigating the finite-sample, finite-SNR performance of the studied method and the relation to the theoretically derived variance. The POWDER method is theoretically investigated for arbitrary array geometries, and all the results hold for general, unambiguous, arrays. Due to the particular appealing form of the estimator implementation when a ULA is employed, we however use such a receiver in the numerical simulations.

The article is structured as follows. In Section 2 we revisit the problem formulation given in [9], and in Section 3 we look at the theoretical derivation of POWDER. We derive an expression for the CRB in Section 4, and we also show how to simplify that expression for high SNR. In Section 5 we make a performance analysis of the POWDER-algorithm and we see that, asymptotically in N and for large SNR, the algorithm obtain the CRB. We conclude the article in Section 6 by numerical Monte-Carlo (MC) experiments, in which we study the performance of the investigated algorithm as well as the applicability of the theoretically derived expression for its accuracy.

Note that we assume, as is common in subspace-based methods, that the number of impinging signals are known, as well as the dimension of the subspace those signals span. See, e.g., [10] for a treatment of the case when such information is unavailable.

We use the following notation: Boldface lowercase (uppercase) letters denote vectors (matrices). The operator $\otimes$ denotes the Kronecker product, and the operator $\mathrm{vec}(\mathbf{X})$ stacks the columns of the matrix $\mathbf{X}$ into a vector. It can be verified that $\mathrm{vec}(\mathbf{ABC}) = \left(\mathbf{C}^{\mathrm{T}} \otimes \mathbf{A}\right)\mathrm{vec}(\mathbf{B})$ for matrices of matching dimensions. The superscripts T, c, and $*$ denote transpose, conjugate, and conjugate transpose, respectively. The notation $\mathrm{Tr}(\mathbf{X})$ denotes the trace of the matrix $\mathbf{X}$, i.e. the sum of the diagonal elements in $\mathbf{X}$. By the symbol $\triangleq$ we indicate a definition. If a quantity $X = O(x)$, then X/x is bounded as $x \to 0$, and if $X = o(x)$, then $X/x \to 0$ as $x \to 0$. We also use $O_p(x)$ and $o_p(x)$, which are the respective in-probability versions [11]. Note that if $X_n = O_p(a_n)$, $Y_n = o_p(b_n)$, then $X_n Y_n = o_p(a_n b_n)$. We denote the Frobenius norm of the matrix $\mathbf{X}$ by $\|\mathbf{X}\|$. We define the projection matrix $\mathbf{\Pi}_{\mathbf{X}} \triangleq \mathbf{X}(\mathbf{X}^*\mathbf{X})^{-1}\mathbf{X}^*$ for any full-rank matrix $\mathbf{X}$, and the orthogonal projector $\mathbf{\Pi}_{\mathbf{X}}^{\perp} \triangleq \mathbf{I} - \mathbf{\Pi}_{\mathbf{X}}$.

2. Problem description

Consider the narrow-band signal model (see e.g. [2])

$$\mathbf{y}(t) = \mathbf{A}(\overline{\boldsymbol{\theta}})\mathbf{x}(t) + \mathbf{n}(t), \quad t = 0, 1, \ldots, N-1. \tag{1}$$

Here, the vector $\mathbf{y}(t) \in \mathbb{C}^{m \times 1}$ represents the sensor array output, and $\mathbf{x}(t) \in \mathbb{C}^{d \times 1}$ the signal samples, at time t. The matrix $\mathbf{A}(\overline{\boldsymbol{\theta}}) \in \mathbb{C}^{m \times d}$ is the array steering matrix, which is uniquely determined by the array geometry and the (assumed distinct) DOAs $\overline{\boldsymbol{\theta}}$ of the impinging signals (we reserve $\boldsymbol{\theta}$ for the *unknown* DOAs, see below). The dimensions m and d correspond to the number of sensors and source signals, respectively. Finally, $\mathbf{n}(t) \in \mathbb{C}^{m \times 1}$ represents the sensor noise. We model both the signal and the noise vectors as zero mean, temporally i.i.d. circularly symmetric complex Gaussian random processes with spatial covariance matrices given by $\mathrm{cov}(\mathbf{x}(t)) = \mathbf{P}$ and $\mathrm{cov}(\mathbf{n}(t)) = \sigma^2\mathbf{I}$, respectively. Using (1) and the definitions above, the sensor output covariance matrix is

$$\mathbf{R} \triangleq \mathrm{E}\left[\mathbf{y}(t)\mathbf{y}^*(t)\right] = \mathbf{APA}^* + \sigma^2\mathbf{I}. \tag{2}$$

The first key assumption in the current article, which delimits it from some well-known state of the art results in this field (e.g. [2]), is that we assume some of the signal directions to be known *a-priori*; hence we are only interested in estimating $d_u = d - d_k$ of the DOAs, where the subscripts u and k henceforth denote unknown and known, respectively. With that fact in mind we can, without loss of generality, write

$$\overline{\boldsymbol{\theta}}_0 = \left[\boldsymbol{\theta}_0^{\mathrm{T}}\ \boldsymbol{\vartheta}^{\mathrm{T}}\right]^{\mathrm{T}}; \tag{3}$$

$$\mathbf{A}(\overline{\boldsymbol{\theta}}_0) = \left[\mathbf{A}(\boldsymbol{\theta}_0)\ \mathbf{A}(\boldsymbol{\vartheta})\right] \triangleq [\mathbf{A}_u\ \mathbf{A}_k]; \tag{4}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_u & \mathbf{P}_{uk} \\ \mathbf{P}_{uk}^* & \mathbf{P}_k \end{bmatrix} \tag{5}$$

where henceforth $\boldsymbol{\theta}_0$ and $\boldsymbol{\vartheta}$ denote the unknown and the known DOAs, respectively. We further distinguish between $\boldsymbol{\theta}_0$, representing the *true values* of the unknown DOAs, and $\boldsymbol{\theta}$, which is the *parametrization* of the unknown DOAs. In (4), the subscripts denote the correlation states between the signals emanating from the unknown and known DOAs. The second defining assumption of this work is

$$\mathbf{P}_{uk} = \mathbf{0}; \tag{6}$$

hence we assume, and exploit, that there is no correlation between the signals from the known and unknown directions. There is no other work known to the authors which have studied this particular instance of the DOA problem; in the DOA scenarios studied in [6] and [12], it was assumed that the source signals were perfectly uncorrelated, i.e. diagonal $\mathbf{P}$ — that assumption is a strict subset of the current assumption $\mathbf{P}_{uk} = \mathbf{0}$. Accordingly, we do not make any assumptions on $\mathbf{P}_k$ or $\mathbf{P}_u$; except, we need to know their respective rank and hence introduce the parameters $d_u' = \mathrm{rank}(\mathbf{P}_u)$, $d_k' = \mathrm{rank}(\mathbf{P}_k)$, and $d' = \mathrm{rank}(\mathbf{P}) = d_u' + d_k'$.