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Blind cluster structured sparse signal recovery: A nonconvex approach

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ABSTRACT

We consider the problem of recovering a sparse signal when its nonzero coefficients tend to cluster into blocks, whose number, dimension and position are unknown. We refer to this problem as *blind cluster structured sparse recovery*. For its solution, differently from the existing methods that consider the problem in a statistical context, we propose a deterministic neighborhood based approach characterized by the use both of a non-convex, nonseparable sparsity inducing function and of a penalized version of the iterative ℓ_1 reweighted method. Despite the high nonconvexity of the approach, a suitable integration of these building elements led to the development of MB-NFCS (*Model Based Nonlinear Filtering for Compressed Sensing*), an iterative fast, self-adaptive, and efficient algorithm that, without requiring any information on the sparsity pattern, adjusts at each iteration the action of the sparsity inducing function in order to strongly encourage the emerging cluster structure. The effectiveness of the proposed approach is demonstrated by a large set of numerical experiments that show the superior performance of MB-NFCS to the state-of-the-art algorithms.

1. Introduction

This paper deals with the problem of recovering a sparse signal when its nonzero coefficients tend to cluster into blocks, whose number, dimension and position are unknown. We refer to this problem as *blind cluster structured sparse recovery*. This kind of sparse pattern is encountered often in practical applications, including gene expression levels, DNA microarrays, MIMO channel equalization, magnetoencephalography, and has recently drawn considerable attention in the Compressed Sensing research area [2,5,19,41].

The basic paradigm of conventional Compressed Sensing is the K-sparsity of an N-dimensional signal. By exploiting this sparsity it is possible to obtain robust signal recovery from $M = O(K \log(N/K))$ linear measurements, by selecting from the signals that agree with the measurements the one with minimum ℓ_1 norm [6–9]. While this result provides a significant improvement over Nyquist-rate sampling, *model based and structured* Compressed Sensing represent new directions of the traditional theory which extend the standard sparsity prior to include structural dependencies between the values and locations of the nonzero signal coefficients [2,5,19,28,41]. Two typical examples of structure are wavelet trees [25] and block-sparsity [19,20]. Structured sparsity models reduce the degrees of freedom of a sparse signal by permitting only certain configurations of the support of the nonzero coefficients. This allows for a higher reduction of the number of measurements required for stable signal recovery. Several papers have been devoted to integrating structured sparsity into Compressed Sensing.

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In [2,19,20] new theoretical results and algorithms are presented for the above-mentioned structure, while [1,3,19,23,24,26,34,36,37,49,50] focus in particular on the block-structure. In these papers several combinations of norms are used to induce both overlapping and non-overlapping group sparsity and different strategies, such as variable splitting and alternating direction method of multipliers (ADMM), are proposed to solve the corresponding convex optimization problem. In the attempt to improve the results based on convex optimization, in [12,52], nonconvex approaches have been proposed for group sparsity with sparse group. All the mentioned methods solve the cluster structured sparse signal recovery problem efficiently, but each of them requires some a priori information about the block partition. The only papers that, to the best of our knowledge, do not require any information on the sparsity structure are [43,48], which present neighborhood based approaches developed in statistical Bayesian frameworks. Their methods, implemented in the Cluss-VB and EBSBL-BO packages, represent the best state-of-the-art algorithms for blind cluster structured sparse signal recovery. The only drawback of these algorithms is that they are computationally demanding.

Motivated by the need to significantly reduce the computing time, this paper focuses on the blind recovery problem with the objective of creating a fast and efficient reconstruction algorithm. We propose two new ideas to achieve this goal: the first is to use in the compressed sensing reconstruction problem, in place of the ℓ_1 norm, a new neighborhood based nonconvex and nonseparable sparsity inducing function, that allows us to capture during the reconstruction process the local interdependency structure of the sparse signal support; the second is to integrate the iterative reweighted ℓ_1 (IRℓ_1) scheme within the iterative penalization approach used for the solution of the corresponding constrained nonconvex minimization problem. Even if the use of a suitable sparsity inducing function to encode prior information on the structure of the signal sparsity is not new [24,29,44], the blind approach is very different. The novelty of our proposal relies on the fact that we do not require any a priori information on the signal structure, since the iterative integration of the penalization and reweighted approach allows the algorithm to discover at each iteration the hidden local structure and suitably adapt the action of the sparsity inducing function by changing the weights accordingly. The resulting MB-NFCS algorithm is thus iterative, and at each iteration of the penalization method the proposed nonseparable sparsity inducing function adaptively encodes the locally emerging cluster structure and encourages the next reconstruction to possess the discovered sparsity pattern. Extensive experiments and comparisons with the best state-of-the-art algorithms for blind structured sparse signal recovery show that the proposed iterative, deterministic approach is very effective, since MB-NFCS is very fast, but still succeeds in achieving perfect reconstruction from a very low number of measurements.

The paper is organized as follows: in Section 2 we briefly recall the basic results of the classical and Model Based Compressed Sensing Theory. The proposed nonconvex approach to the blind cluster structured sparsity problem is presented in Section 3, and in Section 4 we give the solution method and the scheme of the corresponding algorithm. Section 5 presents a brief extension of the proposed approach to the two-dimensional case and in Section 6 we give numerical results demonstrating the effectiveness of the MB-NFCS algorithm. Section 7 closes the paper with a short conclusion.

2. The Compressed Sensing problem: summary of theory

The Compressed Sensing formulation for the problem of recovering a K-sparse N-dimensional signal from M linear measurements $y = \Phi x$, with $M \ll N$ and Φ an $M \times N$ matrix, is the following: find $\hat{u} \in \mathbb{R}^N$ that satisfies:

$$\hat{u} = \arg \min_{u \in \mathbb{R}^N} F(u) \quad \text{subject to } \Phi u = y, \tag{1}$$

where the sparsity inducing function $F(u)$ allows us to select, among the infinitely many solutions of the underdetermined linear system $\Phi u = y$, the desired one. The most natural choice for $F(u)$ is $F(u) = \|u\|_0$, but the corresponding combinatorial search is numerically prohibitive. A common approach is therefore to use $F(u) = \|u\|_1$ as a convex relaxation of the ℓ_0 norm and to solve the corresponding constrained convex optimization problem. Theoretical results have shown the equivalence of the two formulations, provided that the matrix Φ possesses the Restricted Isometry Property of order $2K$ ($2K$-RIP), and the number of measurements M satisfies a Φ-dependent bound of the kind $M = O(K \log(N/K))$ (see [16] and references therein).

The need to further reduce the number of measurements has recently encouraged the emergence of new research directions for the choice of the sparsity inducing function. Among them, *nonconvex* Compressed Sensing is based on a nonconvex approach to problem (1). Allowing the function $F(u)$ to be nonconvex makes it possible to better approximate the ℓ_0-norm, thereby obtaining fast and stable reconstructions with a lower number of measurements [11,30,31,33,35,40,42,45,46]. Another direction known as *model-based* Compressed Sensing accounts for the fact that the nonzero coefficients of some sparse signals tend to cluster into groups or other structured patterns. In order to exploit the interdependency of the nonzero coefficients, model-based Compressed Sensing makes use of suitable sparsity inducing functions $F(u)$ that encode the known sparsity structure, favoring the arrangement of coefficients into the desired pattern. Several papers have been proposed to add the structured sparsity constraint into problem (1), both by using linear combinations of norms on subsets of variables or specifically constructed sparsity inducing functions and by adding suitable constraints on the set of nonzero coefficients (see e.g. [2,18,20,27,29,38]). The main practical interest of inserting structured sparsity constraints into the Compressed Sensing recovery problem is that structured sparsity models reduce the degrees of freedom of sparse signals, thus decreasing the minimal number of measurements M necessary for their stable recovery. In the classical Compressed Sensing approach, an N-dimensional K-sparse signal x lives in $\Sigma_K \subset \mathbb{R}^N$, which is a union of $\binom{N}{K}$ subspaces of dimension K. The only constraint is its sparsity K. By adding further constraints permitting only certain patterns in the support of the signal, we create a structured sparsity model $\mathcal{M}_K$,