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Adaptive frequency estimation of three-phase power systems ☆

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ABSTRACT

The frequency of a three-phase power system can be estimated by identifying the parameter of a second-order autoregressive (AR2) linear predictive model for the complex-valued $\alpha\beta$ signal of the system. Since, in practice, both input and output of the AR2 model are observed with noise, the recursive least-squares (RLS) estimate of the system frequency using this model is biased. We show that the estimation bias can be evaluated and subtracted from the RLS estimate to yield a bias-compensated RLS (BCRLS) estimate if the variance of the noise is known *a priori*. Moreover, in order to simultaneously compensate for the noise on both input and output of the AR2 model, we utilize the concept of total least-square (TLS) estimation and calculate a recursive TLS (RTLS) estimate of the system frequency by employing the inverse power method. Unlike the BCRLS algorithm, the RTLS algorithm does not require the prior knowledge of the noise variance. We prove mean convergence and asymptotic unbiasedness of the BCRLS and RTLS algorithms. Simulation results show that the RTLS algorithm outperforms the RLS and BCRLS algorithms as well as a recently-proposed widely-linear TLS-based algorithm in estimating the frequency of both balanced and unbalanced three-phase power systems.

1. Introduction

In electric power grids, the system frequency normally fluctuates around its nominal value within an acceptable range. Deviation of the system frequency from its nominal range represents an imbalance between load and generation, which is a critical event. Therefore, it is imperative to closely watch the possible variations in the frequency. Most protection-and-control applications in electric power systems require accurate and fast estimation of the system frequency. An erroneous estimate of the frequency may cause a catastrophic grid failure due to inadequate or delayed load shedding [2–7].

In three-phase systems, none of the single phases can faithfully characterize the whole system and its properties. Therefore, a robust frequency estimator should take into account the information of all three phases [8–12]. Clarke's transform applied to the voltages of a three-phase power system produces a complex-valued signal (known as the $\alpha\beta$ signal) that incorporates the information of the three phases. In many applications, the $\alpha\beta$ signal can be considered as a faithful representative for a three-phase system [13]. The phase voltages are digitized at the measurement points by quantizing the samples taken at uniform intervals. Therefore, in practice, the observed voltage data and consequently the $\alpha\beta$ signal are contaminated with noise/error.

From a signal-processing point of view, the noisy samples of the $\alpha\beta$ signal and the sampling rate comprise the available data while the amplitudes of the three phase voltages,

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the initial phase angle, and the system frequency are the unknown parameters. A plethora of techniques have been developed to extract these parameters, particularly the system frequency, from the observable data. Some of the most well-known frequency estimation techniques are based on zero-crossing [14], phase-locked loop [15–17], discrete Fourier transform [18], Viterbi algorithm [19], extended Kalman filter [20], Newton's method [21], data (auto)correlation [22–24], demodulation [25], least-error-squares curve fitting [26], least mean-square (LMS) [27], least mean-phase [28], adaptive notch filters [29,30], and minimum-variance distortionless response (MVDR) spectrum [31].

In order to estimate the system frequency within the realm of linear adaptive signal processing, time evolution of the noiseless $\alpha\beta$ signal can be modeled by either a first-order autoregressive (AR1) [27] or a second-order autoregressive (AR2) linear predictive model [32–36]. The AR1 is suitable for balanced three-phase systems, i.e., when the voltage magnitudes of the three phases are identical. It relates two consecutive noiseless samples of the $\alpha\beta$ signal of a balanced system via a single complex-valued parameter. The modulus of this parameter is equal to unity and its phase angle is equal to the system angular frequency multiplied by the sampling interval. Therefore, the system frequency can be estimated by identifying the parameter of the AR1 model from the noisy voltage observations using any linear estimation technique, e.g., the LMS algorithm as proposed in [27].

The AR1-based frequency estimators lose their accuracy when the system is unbalanced, since in such systems, the strictly-linear AR1 model becomes inexact. Augmenting the AR1 model using the notion of widely-linear modeling [37] can provide a remedy for this weakness in handling the unbalanced three-phase power systems [38–40]. The frequency estimation techniques based on the widely-linear AR1 model can estimate the system frequency by identifying the parameters of the widely-linear model even when the system in unbalanced. However, despite introducing an extra complex-valued model parameter, the widely-linear-AR1-based methods cannot cope with the cases of severe unbalancedness, e.g., when the readings of two phases drop to zero. This is mainly due to the inherent limitation of the widely-linear AR1 model for such events caused by its approximate nature.

On the other hand, the AR2 model linearly relates three consecutive noiseless samples of the $\alpha\beta$ signal via a single real-valued parameter that is equal to the cosine of the product of the system angular frequency and the sampling interval. Thus, the system frequency can be estimated by identifying the parameter of the AR2 model from the noisy observations of the $\alpha\beta$ signal while being harmlessly oblivious to the values of the phase voltage magnitudes and the initial phase angle. In other words, since the parameter of the AR2 model depends only on the system frequency and the sampling interval, any frequency estimator built on this model is virtually insensitive to the balance state of the three-phase power system.

As the $\alpha\beta$ signal is observed with noise, a reliable frequency estimation technique based on the AR2 model should minimize the effect of noise. In [34,35], the least-squares (LS) method has been used for this purpose. A recursive LS (RLS) frequency estimator has also been proposed in [41]. The LS-based approaches are best suited to counter the effect of the noise at the output of a linear model. However, since in the AR2 model, the input of the model is also subject to observational noise, the LS-based frequency estimators are biased. Such bias can falsely indicate a shift in the system frequency and invoke unnecessary corrective actions, which may have harmful fallouts. Thus, any bias in frequency estimation can seriously compromise the stability and reliability of a power system.

One way to eliminate the estimation bias is to evaluate the bias separately and subtract it from the biased estimate [42–44]. The evaluation of the bias usually requires prior knowledge of the noise variance or an extra procedure for estimating the noise variance. Alternatively, the total least-squares (TLS) estimation technique can be utilized to compensate for the noise on both input and output of the AR2 model. TLS is a fitting method that improves accuracy of the LS estimation techniques when both the input and output data of a linear system are subject to observational error. It finds an estimate for the system parameters that fits the input to the output with minimum perturbation in the data. A TLS estimator can eliminate the estimation bias induced by the input noise without performing any explicit bias calculation [45–47]. Two efficient recursive TLS algorithms have been developed in [48,49] utilizing the line-search optimization. The latter minimize a Rayleigh-quotient cost function and the former employs the inverse power method [50]. The TLS technique has recently been utilized to estimate the frequency of three-phase power systems based on the widely-linear AR1 model [51,52].

In this paper, we show that the RLS algorithm based on the AR2 model for the noiseless $\alpha\beta$ signal is biased when applied to adaptive frequency estimation of three-phase power systems at the presence of noise. In order to obtain unbiased estimates while employing the AR2 model, we develop a bias-compensated RLS (BCRLS) algorithm as well as a recursive TLS (RTLS) algorithm. We derive the BCRLS algorithm by evaluating the estimation bias and subtracting it from the biased RLS estimate. To derive the RTLS algorithm, we calculate the TLS estimate of the AR2 model parameter by implementing a single iteration of the inverse power method at each time instant. Unlike the algorithms proposed in [48,49], our RTLS algorithm does not implement any line-search optimization. We show that the BCRLS and RTLS algorithms are convergent in the mean and asymptotically unbiased. We verify the effectiveness of the proposed algorithms in estimating the frequency of both balanced and unbalanced three-phase power systems through simulated experiments.

2. Signal and system model

The phase-to-neutral voltages of a three-phase power system, sampled at the rate of $1/\tau$, are represented by

$$v_{a,n} = V_a \cos(2\pi f \tau n + \theta),$$

$$v_{b,n} = V_b \cos\left(2\pi f \tau n + \theta - \frac{2\pi}{3}\right)$$

and

$$v_{c,n} = V_c \cos\left(2\pi f \tau n + \theta + \frac{2\pi}{3}\right)$$

where f is the system frequency, V_a, V_b, and V_c are the peak voltage values, θ is an initial phase angle, and n is the integer time index. In practice, these voltages are