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A new non-local maximum likelihood estimation method for Rician noise reduction in magnetic resonance images using the Kolmogorov–Smirnov test

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ABSTRACT

Denoising algorithms play an important role in the enhancement of magnetic resonance (MR) images. Effective denoising is vital for proper analysis and accurate quantitative measurements from MR images. Maximum Likelihood (ML) estimation methods were proved to be very effective in denoising MR images. Among the ML based methods, the recently proposed non-local maximum likelihood (NLML) approach gained much attention. In the NLML method, the samples for the ML estimation of the true underlying intensity are selected in a non-local way based on the intensity similarity of the pixel neighborhoods. This similarity is generally measured using the Euclidean distance. A drawback of this approach is the usage of a fixed sample size for the ML estimation resulting in over- or under-smoothing. In this work, we propose an NLML estimation method for denoising MR images in which the samples are selected in an adaptive and statistically supported way using the Kolmogorov–Smirnov (KS) similarity test. The method has been tested both on simulated and real data, showing its effectiveness.

1. Introduction

Noise remains one of the main causes of quality degradation in MRI and is a subject in a large number of papers in the MRI literature. The dominant source of noise in the MR image is the patient's body. The body, being a conductive medium, generates fluctuating fields that will be picked up by the receiver coil [11]. The measurement chain of the MR scanner (coil, electronics, etc.) also contributes to the noise. The signal to noise ratio (SNR) of the image is furthermore influenced by other factors like the strength of the main magnetic field, pulse sequence design, tissue characteristics, RF coil used and imaging parameters like voxel size, number of excitations, receiver bandwidth, etc.

Denoising algorithms play an important role in the enhancement of MR images. Noise in MRI can be naturally minimized by averaging images after multiple acquisitions. This, however, may not be feasible in clinical practice and small animal MR imaging where there is an increasing need for speed. Thus, post-processing techniques to remove noise in the acquired data are important. Also, in time-sensitive acquisitions or studies with limited imaging time (diffusion MRI, functional MRI, etc.), acquisitions with multiple repetitions are not feasible. Many authors applied the conventional classical denoising techniques to denoise MR images [7]. These methods assume the noise in the image to be Gaussian distributed. The major drawback of these methods is that the biasing effects of Rician noise, which characterizes magnitude MR images, are not taken

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into account. This bias increases with decreasing SNR. Later on, many methods were proposed to denoise MR images accounting for this bias. Most of these methods exploited the second moment of the Rice distribution to reduce the bias in the denoised images [9,10]. That is, the image is denoised with the methods based on the Gaussian assumption and to reduce the bias,$2\sigma_g^2$ is subtracted from the squared denoised image (where σ_g^2 is the variance of the noise in the complex image). However, in [17] it was shown that the sample size and SNR have a significant influence on the process of estimating the true underlying signal using this approach. ML methods were proved to be better than the aforementioned methods [8,12]. Very recently an NLM method based on Rician statistics was proposed in [5].

ML based denoising methods applied to magnitude MR images incorporate the Rice distribution to estimate the true underlying signal from a local neighborhood within which the signal is assumed to be constant. However, if this assumption is not met, such filtering will lead to blurred edges and loss of fine structures in the image. As a solution to the blurring issue of the local ML approach, the non-local ML (NLML) estimation method was proposed [8]. The NLML approach was inspired by the work of Buades et al. given in [3]. In the NLML method, the samples for the ML estimation of the true underlying intensity are selected in a non-local (NL) way based on the intensity similarity of the pixel neighborhoods. This similarity is generally measured using the Euclidean distance [8]. In that method, however, the number of NL pixels to be considered for ML estimation is fixed and is determined in a heuristic way. This fixed sample size can introduce under- or over-smoothing in the images. In this work, we propose a non-local ML estimation method for denoising MR images in which the samples are selected in an adaptive way using the Kolmogorov–Smirnov similarity (KS) test.

This paper is organized as follows. Section 2 gives a short overview of the noise characteristics in MRI. Section 3 elaborates the proposed method. Section 4 presents the experimental results, a comparative evaluation and a discussion. Finally, conclusions and remarks are drawn in Section 5.

2. Distribution of magnitude MR data

The acquired complex valued MR signals in the k-space are characterized by a Gaussian probability density function (PDF). The k-space data are then (inverse) Fourier transformed to obtain the magnetization distribution. After the inverse Fourier transform, the real and imaginary components will still be Gaussian distributed due to the linearity and the orthogonality of the Fourier transform. However, due to the subsequent nonlinear transform to a magnitude image, the data will no longer be Gaussian but Rician distributed.

Let R and I represent the real and imaginary parts of the noisy complex MR data (corrupted by zero mean Gaussian, stationary noise with the standard deviation σ_g) with mean values μ_R and μ_I, respectively. Then the reconstructed magnitude data M will be Rician distributed [15]. The corresponding

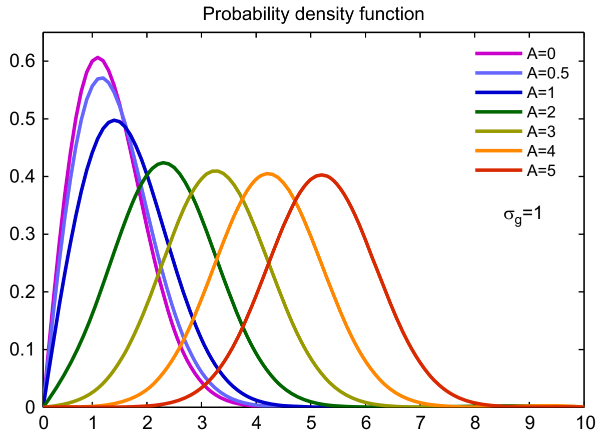

Fig. 1. Rician PDF for different SNR values.

Rician PDF is given by

$$p_{\mathrm{M}}(M|A,\sigma_g) = \frac{M}{\sigma_g^2} e^{-(M^2+A^2)/2\sigma_g^2} I_0\left(\frac{AM}{\sigma_g^2}\right)\varepsilon(M) \tag{1}$$

where $M=\sqrt{R^2+I^2}$, $A=\sqrt{\mu_R^2+\mu_I^2}$, $I_0(.)$ is the 0th order modified Bessel function of the first kind and $\varepsilon(.)$ is the Heaviside step function. The shape of the Rician PDF depends on the signal to noise ratio (SNR), which is here defined as the ratio A/σ_g. Fig. 1 shows the Rice PDF as a function of the magnitude M for various values of the SNR. When $A=0$, the Rice distribution becomes a Rayleigh distribution and the corresponding PDF can be written as

$$p_{\mathrm{M}}(M|\sigma_g) = \frac{M}{\sigma_g^2} e^{-M^2/2\sigma_g^2}\varepsilon(M). \tag{2}$$

In the MR image background, where the SNR is zero due to the lack of water-proton density in the air, the data will follow a Rayleigh distribution. At high SNR, i.e. when $A/\sigma_g \to \infty$, the Rician distribution approaches a Gaussian distribution and the PDF can be written as

$$p_{\mathrm{M}}(M|A,\sigma_g) = \frac{1}{\sigma_g\sqrt{2\pi}} e^{-(M-A)^2/2\sigma_g^2} \tag{3}$$

3. Signal estimation using NLML method

Let $M_1, M_2, \ldots, M_n$ be n statistically independent observations within a region of constant signal intensity A. Then, the joint PDF of the observations is

$$p(\{M_1, M_2, M_3, \ldots, M_n\}|A,\sigma_g) = \prod_{i=1}^{n} \frac{M_i}{\sigma_g^2} e^{-(M_i^2+A^2)/2\sigma_g^2} I_0\left(\frac{AM_i}{\sigma_g^2}\right). \tag{4}$$

The ML estimator $\widehat{A}_{ML}$ of A can now be obtained by maximizing the likelihood function $L(A)$, or equivalently $\ln L(A)$, with respect to A [16]:

$$\widehat{A}_{\mathrm{ML}} = \arg\{\max_A(\ln L)\}, \tag{5}$$