Signal Processing 103 (2014) 92–102

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Compressed sensing by collaborative reconstruction on overcomplete dictionary ☆

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A R T I C L E I N F O

Article history:
Received 9 June 2013
Received in revised form
28 November 2013
Accepted 29 November 2013
Available online 16 December 2013

Keywords:
Collaborative reconstruction
Overcomplete dictionary
Nonlocal self-similarity
Jointly sparse model
Autoregressive model

A B S T R A C T

In this paper, we propose a novel collaborative compressed sensing (CS) reconstruction method for natural images. The method is designed to enhance the accuracy and stability when recovering the sparse representations of image blocks on an overcomplete dictionary from the random measurements by introducing nonlocal self-similarity information. The main idea of collaborative reconstruction is to reconstruct an image block by the collaboration of a group of other blocks sharing similar structures to it, therefore more information is made use for individual blocks than their own measurements. The proposed reconstruction method is composed of two collaborative processes which are derived from two nonlocal self-similarity models: the jointly sparse model and the autoregressive model. By the experimental results, the method is shown to outperform the reconstruction methods without collaborations.

1. Introduction

1.1. Compressed sensing on overcomplete dictionaries

The research of compressed sensing (CS) has attracted great interest by showing the great possibility of surpassing the traditional limits of sampling theory. The extraordinary works by Donoho [1], Candès [2] and Candès et al. [3] show that a signal $\mathbf{x} \in \mathbb{R}^n$ can be reconstructed from a small set of linear and non-adaptive measurements $\mathbf{y} = \mathbf{\Phi x}$ ($\mathbf{y} \in \mathbb{R}^m$, $m \ll n$) by taking advantage of the property of sparsity or compressibility inherent in real world signals. The sparse signal $\mathbf{x}$ can be reconstructed by solving the inverse problem:

$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{s.t. } \mathbf{y} = \mathbf{\Phi x}, \tag{1}$$

where the matrix $\mathbf{\Phi}$ is a random matrix such as Gaussian, Bernoulli, Binary or Fourier matrix which satisfies Restricted Isometry Property (RIP) [3–5] with overwhelming probability. Many signals in real world, natural images for example, are not sparse themselves but sparse in a dictionary: $\mathbf{x} = \mathfrak{D}\mathfrak{s}$, where $\mathfrak{D} \in \mathbb{R}^{n \times N}$ is the sparsifying dictionary and $\mathfrak{s} \in \mathbb{R}^N$ a sparse signal. In image applications, by stacking all the pixels of an image into a column vector $\mathbf{x}$, the reconstruction problem becomes to reconstruct the sparse coefficient vector by solving:

$$\mathfrak{s}^* = \arg\min_{\mathfrak{s}} \|\mathfrak{s}\|_0 \quad \text{s.t. } \mathbf{y} = \mathbf{\Phi}\mathfrak{D}\mathfrak{s}. \tag{2}$$

☆ This work was supported in part by The National Basic Research Program (973 Program) of China (Grant no. 2013CB329402), in part by The National Natural Science Foundation of China (Grant nos. 61072106, 61173090, 61072108, 61362021 and 61303119), in part by The National Research Foundation for the Doctoral Program of Higher Education of China (Grant no. 20110203110006), in part by The Fund for Foreign Scholars in University Research and Teaching Programs (the 111 Project) (Grant no. B07048), in part by The Program for Cheung Kong Scholars and Innovative Research Team in University (No. IRT1170), in part by The Guangxi Natural Science Foundation (Grant no. 2013GXNSFDA019030), and in part by The Fundamental Research Funds for the Central Universities (Grant no. K5051203002).

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http://dx.doi.org/10.1016/j.sigpro.2013.11.039

Then $\mathbf{x}$ is estimated by $\mathfrak{D}\mathfrak{s}^*$. When the number of nonzero components in $\mathfrak{s}$ (also called sparsity) is known to be K, the reconstruction model is given by

$$\mathfrak{s}^* = \arg\min_{\mathfrak{s}} \|\mathbf{y} - \mathbf{\Phi}\mathfrak{D}\mathfrak{s}\|^2 \quad \text{s.t.} \ \|\mathfrak{s}\|_0 \le K. \tag{3}$$

Sparsity is the fundamental prior to solve the ill-posed CS problem. The sparser the signal is, the less the measurement is needed for faithful reconstruction. Orthogonal bases offer simple and fast-implemented decomposing for signals [6], but the representations are not sparse and flexible enough. Overcomplete dictionaries were introduced by [7–9], whose base functions, called atoms, are not necessarily orthogonal to each other. A dictionary matrix has much more columns than rows, i.e. $n \ll N$. Owing to the redundant property of overcomplete dictionaries, signals have more sparse and flexible representations compared to using orthogonal bases. But the flexibility also increases the "illness" of the ill-posed CS problem.

Rewrite the formula of sparse representation by

$$\mathbf{x} = \mathfrak{D}\mathfrak{s} = \sum_{i \in \Lambda} \mathfrak{s}_i \mathbf{d}_i = \mathbf{D}\mathbf{s}, \tag{4}$$

where $\Lambda = \{i | \mathfrak{s}_i \neq 0\}$ is the support of $\mathfrak{s}$, $|\Lambda|$ is the cardinality. In (4), $\mathbf{s} \in \mathbb{R}^K$ keeps the nonzero components of $\mathfrak{s}$ and $\mathbf{D} \in \mathbb{R}^{n \times K}$ is a sub-matrix of $\mathfrak{D}$ composed of those atoms of $\mathfrak{D}$ corresponding to Λ. Substituting (4) for (3), we get the new reconstruction model:

$$(\mathbf{D}^*, \mathbf{s}^*) = \arg\min_{\mathbf{D} \subset \mathfrak{D}, \mathbf{s}} \|\mathbf{y} - \mathbf{\Phi}\mathbf{D}\mathbf{s}\|^2 \quad \text{s.t.} \ \|\mathbf{D}^T\|_{p,0} \le K, \tag{5}$$

where the $l_{p,q}$ norm [10] for matrices is defined as

$$\|\mathbf{A}\|_{p,q} = \left(\sum_i \|\mathbf{a}^i\|_p^q \right)^{1/q}$$

with $\mathbf{a}^i$ denoting the ith row of $\mathbf{A}$. In the special case that $q=0$ for any p, the quasi-norm $\|\mathbf{A}\|_{p,0} = |\text{supp}(\mathbf{A})|$ is used to denote the cardinality of $\text{supp}(\mathbf{A})$ which is the set composed of the indices of the nonzero rows of $\mathbf{A}$. Accordingly, we use $\|\mathbf{D}^T\|_{p,0}$ to denote the number of nonzero columns of $\mathbf{D}$.

In this model the determination of $\mathfrak{s}$ is separated into two parts: the atomic combination $\mathbf{D}$ and the coefficient vector $\mathbf{s}$. Once $\mathbf{D}$ is determined, $\mathbf{s}$ could be determined by the least squared solution:

$$\mathbf{s} = (\mathbf{\Phi}\mathbf{D})^+ \mathbf{y}. \tag{6}$$

Thus the CS reconstruction problem is modeled for choosing an atomic combination $\mathbf{D}$ composed of no more than K atoms out of the dictionary $\mathfrak{D}$ composed of N atoms for sparsely representing the signal $\mathbf{x}$, or equivalently for eliminating the measurement residual $\|\mathbf{y} - \mathbf{\Phi}\mathbf{D}\mathbf{s}\|^2$.

The optimal solution of (5) is not unique and the problem of choosing atoms is a combinatorial problem with NP-hard computational complexity. It is hard to get any one of the solutions. On the other hand, solutions that gain small residuals do not necessarily correspond to meaningful images. Accurate reconstruction of images cannot be achieved by only making use of sparsity prior. More prior knowledge about natural images needs to be discovered and incorporated.

1.2. Collaborative reconstruction for block CS

The block CS framework [11] and the block compressed sampling method are adopted to acquire the block-based measurements of an image. To introduce sparsity for blocks, we construct a Ridgelet overcomplete dictionary. In such a scenario, the reconstruction of an image aims to recover the sparse representations of its blocks on the dictionary from the measurements.

For each block, the ill-posed reconstruction problem is of the form of (3) or (5). The resulted solution is usually inaccurate and unstable by only using its own measurement which is the convolution of the block and the compressed sampling operator. To enhance the accuracy and stability of the solutions of individual blocks, we propose the collaborative reconstruction method which is derived from the nonlocal self-similarity property of natural images.

The main idea of collaborative reconstruction is to determine the atomic combination for an image block not only by the measurement of itself, but also by the collaboration of a group of other blocks sharing similar structures to it. By the collaborative models, the information and constrains for individual image blocks are increased, while the uncertainty and the degree of freedom of the reconstruction problems are decreased.

Fig. 1 is the diagram of the block-based CS and the proposed collaborative reconstruction method. The proposed method is composed of two successive collaborative processes. In the first process, a collaborative model guided by the jointly sparse model is established. An image block is estimated by the measurement of itself and a group of its nonlocal neighbors. Based on the results of the first process, a collaborative model guided by the autoregressive (AR) model is established in the second process. The solution of an image block is refined by the collaboration of its local and nonlocal neighbors.

1.3. Related works

Nonlocal self-similarity models have found their widespread and successful applications in image enhancement and restoration e.g. de-noising [12]. According to the property, a local structure of an image often shows up in

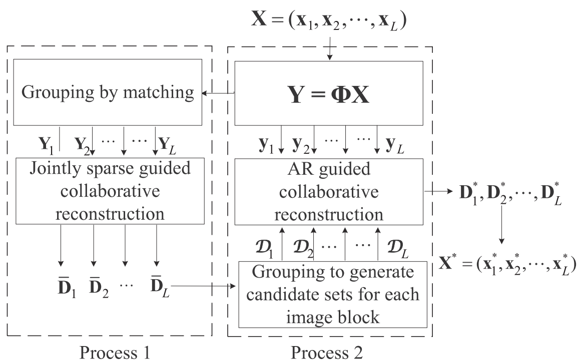

Fig. 1. The diagram of the collaborative reconstruction method.