Signal Processing 103 (2014) 348–366

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

The novel two-dimensional adaptive filter algorithms with the performance analysis

Mohammad Shams Esfand Abadi *, Sahar Nikbakht Aali

Shahid Rajaee Teacher Training University, Faculty of Electrical and Computer Engineering, P.O. Box 16785-163, Tehran, Iran

ARTICLE INFO

Article history:
Received 9 June 2013
Received in revised form
27 November 2013
Accepted 15 December 2013
Available online 31 December 2013

Keywords:
Two-dimensional adaptive filters
Selective partial update
Selective regressor
Mean-square performance
2D system identification
Noise cancellation

ABSTRACT

Two-dimensional (2D) adaptive filtering is a technique that can be applied to many images, and signal processing applications. This paper extends the one-dimensional adaptive filter algorithms to 2D structures and the novel 2D adaptive filters are established. Based on this extension, the 2D selective partial update NLMS (2D-SPU-NLMS), the 2D selective partial update APA (2D-SPU-APA) and the 2D selective regressor APA (2D-SR-APA) are presented. In 2D-SPU adaptive algorithms, the filter coefficients are partially updated, and in 2D-SR-APA, the recent regressors of input signal are optimally selected in each time iteration. These algorithms reduce the computational complexity in 2D adaptive filter applications. In the following, a unified approach for the establishment and mean-square performance analysis of the family of 2D adaptive filter algorithms is presented. This analysis is based on energy conservation arguments and does not need to assume a Gaussian or white distribution for the regressors. We demonstrate the good performance of the proposed algorithms through several simulation results in 2D system identification and 2D adaptive noise cancellation (2D-ANC) for image restoration. The results are compared with the classical 2D adaptive filters such as 2D-LMS, 2D-NLMS, and 2D-APA. Also we show that the derived theoretical expressions are useful in predicting the steady-state and transient performance of 2D adaptive filter algorithms.

1. Introduction

Two-dimensional (2D) adaptive filters as well as one-dimensional adaptive filters have received a great deal of attention in the last two decades [1], and that is because of their ability to take into account the inherent nonstationary statistical properties of two-dimensional data, as well as 2D statistical correlation. The 2D adaptive filters have been applied to a variety of image processing applications such as image restoration, image enhancement, adaptive noise cancellation, 2D adaptive line enhancer, and 2D system identification. In [2], the one-dimensional least mean squares (LMS) adaptive algorithm was extended to the 2D application and this algorithm was used for estimation of nonstationary images. In [3], an algorithm was proposed in which the McClellan transformation was used. The new 2D-LMS whose convergence properties were not restricted to one direction was proposed in [4]. Also, the development of a 2D adaptive filter using the block diagonal LMS method was presented in [5]. The 2D-LMS adaptive filter [2] is essentially an extension of its one-dimensional counterpart. Its input, like other 2D algorithms, is a two-dimensional array of pixel values. Even though the 2D-LMS is an attractive adaptation algorithm because of its simple structure, it is highly sensitive to eigenvalue disparity, and its convergence speed is slow which is not appropriate in many applications. Therefore,

* Corresponding author.
Tel./fax: +98 21 22970003.
E-mail addresses: mshams@srttu.edu (M. Shams Esfand Abadi), sahar.nikbakht@elec.iust.ac.ir (S. Nikbakht Aali).

0165-1684/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sigpro.2013.12.016

to overcome this problem, the 2D normalized LMS (2D-NLMS) algorithm was proposed. In this algorithm, the influence of magnitude of the filter input on the convergence speed was considered. The 2D adaptive FIR filter which was based on affine projection algorithm (2D-APA) was firstly introduced in [1]. In this algorithm, the positions of projection vectors can be selected freely and its performance is improved especially when the input data is highly correlated. In this case, the performance of 2D-LMS type algorithms becomes deteriorated, because their 2D autocorrelation matrix has large eigenvalues. Unfortunately, this improvement comes at the expense of a higher computational complexity. In [6], a fast affine projection algorithm for two-dimensional adaptive linear filtering was presented. The results show that this algorithm has fast convergence speed and good tracking ability. The 2D recursive least squares (2D-RLS) algorithm [7–9] was proposed before. The computational complexity of 1D-RLS is high by itself and extending it to 2D version can increase the computational complexity.

In the classical adaptive filters, the filter coefficients are fully updated. To reduce the computational complexity, other adaptive filter algorithms were introduced where the coefficients are partially updated. In these algorithms, the filter coefficients which should be updated are optimally selected during the adaptation. The Max-NLMS [10], the *N*-Max NLMS [11,12] (*N* is the number of filter coefficient to update), the variants of the selective partial update NLMS (SPU-NLMS) [13–15], selective partial APA (SPU-APA) [14] and selective regressor APA (SR-APA) [16] are important examples of these adaptive filters.

As computational complexity is one of the most important problems of 2D adaptive filters and no 2D algorithms have so far been introduced to reduce the computational cost, the new algorithms named 2D-SPU-NLMS, 2D-SPU-APA, and 2D-SR-APA are proposed to reduce the complexity in 2D adaptive filter applications. In 2D-SPU-NLMS and 2D-SPU-APA, the filter coefficients are partially updated and in 2D-SR-APA, the number of recent regressors is optimally selected during the filter coefficients update equation. These new algorithms have also close convergence speed to classical 2D adaptive algorithms.

In contrast to one-dimensional adaptive filter algorithm, the theoretical performance analysis of 2D adaptive filter algorithms has not been widely studied. In this paper, we present a unified approach to mean-square performance analysis of the presented 2D adaptive filters as well as the classical adaptive algorithms. Based on this approach, the general expressions are derived, and the transient and steady-state performance of 2D adaptive filters can be studied. This analysis is based on energy conservation arguments and does not need to assume the Gaussian or white distribution for the regressors [17–19].

Also, the parameters selection of many 2D adaptive filter algorithms was not considered completely. Many of these parameters have been selected by trial and error approach in different literatures. In this paper, we study the former and new 2D adaptive algorithms comprehensively. The performance of the presented algorithms is investigated for two more important applications of 2D adaptive filters, namely 2D system identification and 2D adaptive noise cancellation (2D-ANC) for restoration of original images from noisy images.

What we propose in this paper can be summarized as follows:

- The establishment of the novel 2D adaptive filters such as 2D-SPU-NLMS, 2D-SPU-APA, and 2D-SR-APA which lead to the reduction of the computational complexity in 2D applications.
- The mean-square performance analysis of 2D adaptive filter algorithms and studying the transient and steady-state performance of them.
- Demonstration of the presented algorithms in 2D system identification and 2D-ANC applications and the validity of our approach for the mean-square and steady-state performance of 2D adaptive filters in 2D system identification setup.

This paper is organized as follows. Section 2 reviewed the classical 2D adaptive filter algorithms. In Section 3, the novel 2D adaptive filter algorithms are established. The computational complexity of the derived algorithms is presented in Section 4. In Section 5, the general mean square performance analysis for the family of 2D adaptive algorithms is developed and the expressions for the theoretical learning curves, the mean-square coefficient deviation (MSD), and the steady-state mean-square error (MSE) are derived. We conclude the paper by showing a comprehensive set of simulation results demonstrating the usefulness of our results.

Throughout the paper, the following notations are adopted:

$(\cdot)^T$	transpose of vector or a matrix
Tr(.)	trace of a matrix
$\|\cdot\|^2$	squared Euclidean norm of a vector
$\|\cdot\|_F$	Frobenius norm
$\lvert . \rvert$	absolute value of a scalar
$E[\cdot]$	expectation operator
$\|\mathbf{t}\|^2_{\boldsymbol{\Sigma}}$	$\boldsymbol{\Sigma}-$weighted Euclidean norm of a column vector $\mathbf{t}$ defined as $\mathbf{t}^T\boldsymbol{\Sigma}\mathbf{t}$
vec($\mathbf{T}$)	creating an $\mathbf{M}^2\times 1$ column vector through stacking the columns of the $\mathbf{M}\times\mathbf{M}$ matrix $\mathbf{T}$
vec($\mathbf{t}$)	creating an $\mathbf{M}\times\mathbf{M}$ matrix $\mathbf{T}$ from the $\mathbf{M}^2\times 1$ column vector $\mathbf{t}$
$\mathbf{C}\otimes\mathbf{D}$	Kronecker product of matrices $\mathbf{C}$ and $\mathbf{D}$

2. Background on classical 2D adaptive filter algorithms

2.1. 2D-LMS adaptive filter algorithm

Linear system parameterization is an important class of system modeling with a wide area of applications. The most popular among the class of linear model is the finite impulse response (FIR). It is proposed in order to simplify the estimation task and to reduce the computational load in real-time application [6]. Let it be the input of a linear 2D FIR model, defined over a regularly spaced lattice: $(i,j)\in[M_1,M_2]$, where