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Delay-dependent H_∞ filtering for singular Markovian jump time-delay systems

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ABSTRACT

The problem of delay-dependent H_∞ filtering is investigated for a kind of singular Markovian jump time-delay systems in this paper. Without performing the free-weighting matrices method, a delay-dependent bounded real lemma (BRL) is given ensuring the singular system achieves mean-square exponentially admissible and guarantees a prescribed H_∞ performance index in terms of linear matrix inequality (LMI) approach. Based on the BRL, the H_∞ filtering problem is solved and the desired filter can be constructed by solving the corresponding LMIs. Some numerical examples are given to show the effectiveness and the potential of the proposed techniques.

1. Introduction

As an important kind of hybrid systems, Markovian jump systems have received increasing attention in the past few years due to the fact that they have strong practical relevance in mechanical systems, economics systems with human operators, and other engineering areas [1–5]. Recently, considerable attention has been focused on the H_∞ filtering problem for Markovian jump time-delay systems. Among these references, we note that the H_∞ filtering problem was discussed for linear Markovian jump time-delay systems in [6–10] via linear matrix inequality (LMI) approach. When non-linear disturbances appear, a delay-dependent approach was developed to deal with the H_∞ filtering problem for a kind of Itô type stochastic Markovian jump time-delay systems in [11] and a full-order H_∞ filter was designed. It is worth pointing out that compared with traditional Kalman filtering, the H_∞ filtering approach does not require knowledge of the statistical properties of the external noises, which makes the H_∞ filtering approach useful in many applications [12].

On the other hand, singular systems have extensive applications in electrical circuits, power systems, economics and other areas [13–15]. Singular systems are also referred to as generalized systems, descriptor systems, implicit systems, differential-algebraic systems or semi-state systems. Many results have been established for singular systems with or without time-delay; see, e.g. [16–23] and the references therein. Very recently, some attention has been paid to the H_∞ filtering problem for singular systems with or without time delay, see, for example, [23–29] and the reference therein. When Markovian jumping parameters appear, [30] investigated the robust H_∞ filtering problem for mode-dependent time-delay discrete Markov jump singular systems with parameter uncertainties, and a Markov jump filter was designed guaranteeing the filtering error system to be regular, causal, stochastically stable and satisfy H_∞ performance for all admissible uncertainties. While [15,31] discussed the H_∞ filtering problem for continuous singular Markovian jumping systems without time delay and singular Markovian jumping systems with time delay, respectively. However, the results of [31] are delay-independent, and the involved time-delays are

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time-invariant, which limits the scope of applications of the robust H_∞ filtering results. It is well known that delay-dependent conditions are generally less conservative than delay-independent ones, especially when the size of the time delay is small. To the best of our knowledge, the problem of delay-dependent H_∞ filter design for continuous singular systems with both Markovian jump parameters and time delays has not been fully investigated. On the other hand, it should be pointed out that most results on the delay-dependent H_∞ (or $\mathcal{L}_2-\mathcal{L}_\infty$) filter design for all sorts of time-delay systems is based on the free-weighting matrices method [32–36], see, for example, [7,8,30,35–42] and the reference therein. The free-weighting matrices method can effectively reduce the conservatism of the proposed delay-dependent results, however, some researchers have also pointed out a chief shortcoming of such method is that too many free-weighting matrices introduced in the theoretical derivation sometimes cannot reduce the conservatism of the obtained results, on the contrary, some of them make the system analysis and synthesis complicated [43]. How to overcome the aforementioned disadvantage of the free-weighting matrices method is an important research topic in the delay-dependent related problem and also motivates the work of this paper.

This paper deals with the problem of delay-dependent H_∞ filtering for singular systems with both time varying delay and Markovian jump parameters. A bounded real lemma (BRL) is proposed to guarantee the considered system to be delay-dependent exponentially admissible in mean-square and satisfy a prescribed H_∞ performance level. Based on this, an LMI-based approach is proposed to design the desired filters. Finally, some illustrative examples are provided to demonstrate the effectiveness of the proposed methods.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices. $\|\cdot\|$ stands for the Euclidean norm for a vector and $C_{n,d}=C([-d,0],\mathbb{R}^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d,0]$ into $\mathbb{R}^n$ with norm $\|\phi(t)\|_d=\sup_{-d\le s\le 0}\|\phi(s)\|$. $\mathcal{L}_2[0,\infty)$ stands for the space of square integrable functions on $[0,\infty)$. $(\Omega,\mathcal{F},\mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the $\sigma-$algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. The superscript "T" and + represent the transpose and the Moore–Penrose inverse, respectively, and "$*$" denotes the term that is induced by symmetry.

2. Definitions and problem formulation

Let $\{r_t, t\ge 0\}$ be a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S}=\{1,2,\ldots,s\}$ with transition probability matrix $\Pi\triangleq\{\pi_{ij}\}$ given by

$$\Pr\{r_{t+h}=j|r_t=i\}=\begin{cases}\pi_{ij}h+o(h), & j\neq i,\\ 1+\pi_{ii}h+o(h), & j=i,\end{cases}$$

where $h>0$, $\lim_{h\to 0}o(h)/h=0$, and $\pi_{ij}\ge 0$, for $j\neq i$, is the transition rate from mode i at time t to mode j at time $t+h$ and $\pi_{ii}=-\sum_{j=1,j\neq i}^{s}\pi_{ij}$.

Fix a probability space $(\Omega,\mathcal{F},\mathcal{P})$ and consider the singular time-delay systems with Markovian jump parameters

$$\begin{aligned}
E\dot{x}(t)&=A(r_t)x(t)+A_d(r_t)x(t-d(t))+B_\omega(r_t)\omega(t),\\
y(t)&=C(r_t)x(t)+C_d(r_t)x(t-d(t))+D_\omega(r_t)\omega(t),\\
z(t)&=L(r_t)x(t),\\
x(t)&=\phi(t),\quad t\in[-d_2,0],
\end{aligned}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $y(t)\in\mathbb{R}^s$ is the measurement, $z(t)\in\mathbb{R}^q$ is the signal to be estimated and $\omega(t)\in\mathbb{R}^p$ is the disturbance input that belongs to $\mathcal{L}_2[0,\infty)$. $\phi(t)\in C_{n,d_2}$ is a compatible vector valued initial function. The matrix $E\in\mathbb{R}^{n\times n}$ may be singular and it is assumed that rank $E=r\le n$. $A(r_t)$, $A_d(r_t)$, $B_\omega(r_t)$, $C(r_t)$, $C_d(r_t)$, $D_\omega(r_t)$ and $L(r_t)$ are known real constant matrices with appropriate dimensions for each $r_t\in\mathcal{S}$. $d(t)$ is a time-varying continuous function that satisfies

$$0\le d_1\le d(t)\le d_2,\quad \dot{d}(t)\le\mu,\tag{2}$$

where d_1 and d_2 are the time delay lower and upper bounds, respectively, and $0\le\mu<1$ is the time delay variation rate.

Throughout the paper we shall use the following concepts.

Definition 1 (*Yue and Han [44], Wu et al. [45], Boukas et al. [46]*). 1. The singular Markovian jump time-delay system

$$\begin{aligned}
E\dot{x}(t)&=A(r_t)x(t)+A_d(r_t)x(t-d(t)),\\
x(t)&=\phi(t),\quad t\in[-d_2,0]
\end{aligned}\tag{3}$$

is said to be regular and impulse free for any time delay $d(t)$ satisfying (2), if the pairs $(E,A(r_t))$ and $(E,A(r_t)+A_d(r_t))$ are regular and impulse free for every $r_t\in\mathcal{S}$.

2. The singular Markovian jump time-delay system (3) is said to be mean-square exponentially stable, if there exist scalars $\alpha>0$ and $\beta>0$ such that $\mathcal{E}\{\|x(t)\|^2\}\le\alpha e^{-\beta t}\|\phi(t)\|_{d_2}^2$, $t>0$.

3. The singular Markovian jump time-delay system (3) is said to be mean-square exponentially admissible, if it is regular, impulse free and mean-square exponentially stable.

Definition 2. Given a scalar $\gamma>0$, the singular Markovian jump time-delay system (1) is said to be mean-square exponentially admissible with H_∞ performance γ, if the system with $\omega(t)\equiv 0$ is mean-square exponentially admissible and, under zero initial condition, it satisfies $\|z(t)\|_{E_2}<\gamma\|\omega(t)\|_2$ for any non-zero $\omega(t)\in\mathcal{L}_2[0,\infty)$, where

$$\|z(t)\|_{E_2}=\sqrt{\mathcal{E}\left\{\int_0^\infty z(t)^{\mathrm{T}}z(t)\,\mathrm{d}t\right\}}.$$

In this paper, in order to estimate $z(t)$, we are interested in designing a filter of the following structure:

$$\begin{aligned}
E_f\dot{\hat{x}}(t)&=A_f(r_t)\hat{x}(t)+B_f(r_t)y(t),\\
\hat{z}(t)&=C_f(r_t)\hat{x}(t),
\end{aligned}\tag{4}$$