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Fast communication

A variable-step-size NLMS algorithm using statistics of channel response

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ABSTRACT

This paper proposes a new variable-step-size control for the normalized least-mean-square (NLMS) algorithm. A step-size vector is used, with a different value for each adaptive weight. With prior knowledge of the channel impulse response statistics, the optimal step-size vector is obtained by minimizing the mean-square deviation (MSD) between the optimal and estimated filter coefficients. In addition, filter convergence is proved and the relationship between the proposed and existing algorithms are analyzed. The proposed method achieves better steady-state performance compared to existing algorithms. The effectiveness of the proposed algorithm is demonstrated through computer simulations.

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1. Introduction

Adaptive filters have been widely adopted by various applications such as acoustic communications, speech recognition, radar systems, seismology, and biomedical engineering. In the literature, the normalized least-mean-square (NLMS) algorithm is well received for its simplicity and robustness [1]. A critical parameter is the step size, which controls convergence rate, mean-square error (MSE), and computational cost of the NLMS algorithm [2]. To balance the trade-off among different aspects of adaptive filtering, an appropriate step-size control is needed. In the literature, a number of variable-step-size (VSS) NLMS algorithms have been proposed (see e.g., [2–4] and references therein). For instance, in [3], a nonparametric VSS-NLMS (NPVSS-NLMS) algorithm is proposed and the step size is derived by correlating the so-called *a priori* and *a posteriori* error signals. In [4], the step size is obtained by minimizing the mean-square deviation (MSD), and becomes more attractive in certain applications due to its faster convergence rate.

Recently, the impulse response statistics are suggested as one essential factor that facilitates the performance of adaptive filters [5,6]. For applications where the unknown impulse response exhibits a constant exponential decay envelope, such as for room impulse responses, [5] proposed an exponentially weighed step-size (ES) control algorithm to improve the convergence rate. Instead of using a fixed step-size vector as in [5], a variable step-size vector (VSSV) NLMS algorithm is proposed in [6] to achieve both fast convergence and low steady-state error. Moreover, in [7,8], variable tap length (VTL) is considered together with step size to improve the filter performance.

In this paper, a new VSS control is proposed based on the channel response statistics. Similar to [6], a vector step size is used rather than a scalar one as in traditional methods. The step-size vector is derived by minimizing the MSD of filter taps and a novel step-size adaptation scheme is proposed. The proposed method is compared with the existing ones and the convergence is analyzed. We also show that the proposed algorithm improves the steady-state performance compared to some existing

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methods, including: traditional NLMS [1], ES [5], Shin's algorithm [4], and VSSV NLMS [6].

2. Variable-step-size NLMS algorithm

In this section, we describe the model, derive the proposed VSS NLMS algorithm, perform convergence analysis, compare it with some existing methods, and analyze the computational complexity.

2.1. Problem formulation

Consider a linear system with its input signal $x(n)$ and output signal $d(n)$ related by

$$d(n)=\boldsymbol{h}^T\boldsymbol{x}(n)+\nu(n), \tag{1}$$

where $\boldsymbol{h}=[h_0,\ldots,h_{L-1}]^T$ is the unknown system with memory length L, $\boldsymbol{x}(n)=[x(n),\ldots,x(n-L+1)]^T$ is the system input vector, and $\nu(n)$ is additive noise. Here we consider adaptive estimation of h_i, i=0,…,$L-1$, given $d(n)$ and $x(n)$, using the NLMS algorithm. Define the adaptive filter coefficient vector $\boldsymbol{w}(n)=[w_0(n),\ldots,w_{L-1}(n)]^T$. The NLMS coefficients are updated by

$$\boldsymbol{w}(n+1)=\boldsymbol{w}(n)+\mathbf{U}(n)\frac{\boldsymbol{x}(n)e(n)}{\|\boldsymbol{x}(n)\|_2^2}, \tag{2}$$

where $e(n)$ is the estimation error defined as

$$e(n)=d(n)-\boldsymbol{w}^T(n)\boldsymbol{x}(n), \tag{3}$$

the step-size matrix $\mathbf{U}(n)=\mathrm{diag}[\mu_0(n),\ldots,\mu_{L-1}(n)]$, diag[·] denotes a diagonal matrix, and $\|\cdot\|_2$ denotes the l_2 norm. A slight difference between (2) and the traditional NLMS algorithm [1] is that for each filter coefficient $w_i(n)$, a different variable step size $\mu_i(n)$ is applied instead of a common one. The purpose is to enhance the convergence rate and reduce the MSD or MSE of the adaptive filter. Thus, the objective of this paper is to design the step size matrix $\mathbf{U}(n)$ to improve the NLMS algorithm performance.

2.2. Derivation of the optimum step size

Define the coefficient error vector

$$\boldsymbol{\delta}(n)=\boldsymbol{w}(n)-\boldsymbol{h}. \tag{4}$$

Combining (1) and (4), we rewrite the signal estimation error in (3) as

$$e(n)=-\boldsymbol{\delta}^T(n)\boldsymbol{x}(n)+\nu(n). \tag{5}$$

Substituting (5) into (2), we obtain

$$\boldsymbol{\delta}(n+1)=\mathbf{A}(n)\boldsymbol{\delta}(n)+\mathbf{U}(n)\frac{\boldsymbol{x}(n)\nu(n)}{\|\boldsymbol{x}(n)\|_2^2}, \tag{6}$$

where

$$\mathbf{A}(n)=\boldsymbol{I}_L-\mathbf{U}(n)\frac{\boldsymbol{x}(n)\boldsymbol{x}^T(n)}{\|\boldsymbol{x}(n)\|_2^2}, \tag{7}$$

and $\boldsymbol{I}_L$ is the $L\times L$ identity matrix.

To quantitatively evaluate the misadjustment of the filter coefficients, the MSD is taken as a figure of merit, which is defined as

$$\Lambda(n)=E\left[\|\boldsymbol{\delta}(n)\|_2^2\right], \tag{8}$$

where $E[\cdot]$ denotes the statistical expectation. Note that at each iteration, the MSD depends on $\mu_i(n)$. Combining (6) and (8), we obtain

$$\Lambda(n+1)=E\left[\boldsymbol{\delta}^T(n)\mathbf{A}^T(n)\mathbf{A}(n)\boldsymbol{\delta}(n)\right]+\gamma, \tag{9}$$

where

$$\gamma=\mathrm{tr}\left[\mathbf{U}^2(n)\right]\sigma_\nu^2/(L^2\sigma_x^2), \tag{10}$$

and tr[·] denotes the trace of a matrix. Similar to [5,6], we assume that $x(n)$ and $\nu(n)$ are zero-mean i.i.d. stationary with variance σ_x^2 and σ_ν^2, respectively; $\boldsymbol{\delta}(n)$, $x(n)$, and $\nu(n)$ are mutually independent; and $\boldsymbol{x}^T(n)\boldsymbol{x}(n)\approx L\sigma_x^2$ for $L\gg 1$. We obtain

$$E[\mathbf{A}^T(n)\mathbf{A}(n)]=\left\{1+\frac{\mathrm{tr}[\mathbf{U}^2(n)]}{L^2}\right\}\boldsymbol{I}_L-\frac{2}{L}\mathbf{U}(n). \tag{11}$$

Combining (11) and (9), we get

$$\Lambda(n+1)=\left\{1+\frac{\mathrm{tr}\left[\mathbf{U}^2(n)\right]}{L^2}\right\}E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]-\frac{2}{L}E\left[\boldsymbol{\delta}^T(n)\mathbf{U}(n)\boldsymbol{\delta}(n)\right]+\gamma. \tag{12}$$

The optimal step size is obtained by minimizing the MSD at each iteration. Taking the first-order partial derivative of $\Lambda(n+1)$ with respect to $\mu_i(n)$ $(i=0,\ldots,L-1)$, and setting it to zero, we obtain

$$\mu_i(n)=\frac{LE\left[\delta_i^2(n)\right]}{E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]+\sigma_\nu^2/\sigma_x^2}. \tag{13}$$

To update $\mu_i(n)$, we rewrite (13) as

$$\mu_i(n)=\frac{LE\left[\delta_i^2(n)\right]}{E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]}\cdot\frac{\sigma_x^2E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]}{\sigma_x^2E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]+\sigma_\nu^2}. \tag{14}$$

Note that the first term in (14) contains the directional information for the step-size vector update, and the second term represents its magnitude, which depends on the system condition i.e., signal-to-noise ratio (SNR) and filter convergence status in terms of MSD. To update $E[\delta_i^2(n)]$, we use the following equation obtained by taking the mean square of the i th entry in (6):

$$E\left[\delta_i^2(n+1)\right]=\left[1-\frac{2\mu_i(n)}{L}\right]E\left[\delta_i^2(n)\right]+\frac{\mu_i^2(n)}{L^2}E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]+\frac{\mu_i^2(n)\sigma_\nu^2}{L^2\sigma_x^2}. \tag{15}$$

Combining (1), (3), and (4), we obtain

$$E[e^2(n)]=\sigma_x^2E\left[\|\boldsymbol{\delta}(n)\|_2^2\right]+\sigma_\nu^2. \tag{16}$$

Substitution of (16) into (15) leads to

$$E\left[\delta_i^2(n+1)\right]=\left[1-\frac{2\mu_i(n)}{L}\right]E\left[\delta_i^2(n)\right]+\frac{\mu_i^2(n)}{L^2\sigma_x^2}E\left[e^2(n)\right]. \tag{17}$$

It is straightforward to estimate $E[e^2(n)]$ by a moving average of $e^2(n)$:

$$\hat{\sigma}_e^2(n)=\lambda\hat{\sigma}_e^2(n-1)+(1-\lambda)e^2(n), \tag{18}$$