Signal Processing 96 (2014) 241–252

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Relative phase in dual tree shearlets

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ARTICLE INFO

Article history:
Received 7 March 2013
Received in revised form
22 July 2013
Accepted 11 September 2013
Available online 7 October 2013

Keywords:
Shearlets
Dual tree
Uniform downsampling
Complex wavelet
Relative phase
Texture retrieval

ABSTRACT

The relative phase is an efficient approach to exploit the phase information of complex wavelet coefficients. However, the relative phase original generated from the pyramidal dual tree directional filter bank (PDTDFB) has three defaults. Firstly, its texture retrieval performance does not simultaneously improve in general as the scale increases. Secondly, it is not accurate enough when the directional subbands are not uniformly downsampled along row and column. Thirdly, its 2^n number of directions are not optimal. In this paper, we propose a new multiscale and multidirection transform for relative phase, named as dual tree shearlets. The transform is based on the discrete shearlet transform, but a dual tree Laplacian pyramid is adopted to create a real-imaginary pair structure for deriving phase information under multiscale framework. The dual tree shearlets have the properties of uniform downsampled subbands; higher directional sensitivity and the 2-D Hilbert transform relationship between two channels like the dual tree complex wavelet transform (DTCWT). The numerical experiments presented in this paper demonstrate that the relative phase of our proposed method outperforms that of the PDTDFB in texture retrieval application both in terms of performance and computational efficiency. The results show that relative phase of the dual tree shearlets amends the above mentioned defaults.

1. Introduction

For image compression or content-based image retrieval, a rich, reliable and precise representation of the location of features is essential. Wavelet transforms have emerged as a popular basis for image content analysis due to their ability of isolating image energy concisely into scalar, directional and spatial. Compared with conventional real-valued wavelets, complex wavelets which are able to provide both magnitude and phase information have shown a consistent representation to the structures in images. The magnitudes of complex wavelet coefficients indicate the amplitude of features, and the phases indicate the locations of these features.

In the earlier research, through the example of reconstructing a signal with phase of the Fourier coefficients alone, Oppenheim and Lim present the importance of phase information in [1]. Then Morrone and R.A. Owens [2] further demonstrate that phase information is a crucial component in representing the structures of image. Afterwards, Gabor phases have been used in iris and palm-print identification [3,4], face recognition [5] and texture discrimination [6]. Some other applications suggest the relationships of interscale phase in complex wavelet domain, such as texture synthesis [7], blurred detection [8] and face recognition [9].

More recent studies concentrate on exploiting the phase information of complex wavelet coefficients. Suggesting the use of a modified product of coefficients at the same location and adjacent scales in dual tree complex wavelet transform (DTCWT) [10,11]. M. Miller and N. Kingsbury [12] provide an approach to capture local phase relationship across scale and spaces. Consequently,

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0165-1684/$ - see front matter
http://dx.doi.org/10.1016/j.sigpro.2013.09.020

an efficiently denoising algorithm using the modified coefficients according to [12] has been presented in [13]. Another method investigated in [14] attempts to utilize the difference of phase between adjacent complex wavelet coefficients namely relative phase for texture image retrieval. Then in [15], the feature orientation of all subbands is linearly proportional to the relative phase in the pyramidal dual tree directional filter bank (PDTDFB) [16], DTCWT [10,11] and uniform discrete curvelet transform (UDCT) [17]. The probability density function of relative phase is proposed in [18].

As a complementary to the magnitude, the relative phase information of complex wavelet coefficients has been studied to obtain new features in texture image retrieval when images are decomposed by different complex wavelet transforms including PDTDFB and DTCWT. However, through further research we found that the relative phase original generated from PDTDFB has the following defaults. Firstly, the retrieval accuracy would not be simultaneously higher in general as the scale and direction increases as shown in Tables 1 and 2 if the features are derived only from the relative phase. Thus it cannot fully utilize the multiscale information though the relative phase is originally defined in the multiscale framework. Secondly, the relative phase is not accurate enough when the directional subbands are obtained through nonuniform downsampling along row and column respectively. Moreover, PDTDFB and DTCWT can capture the geometry of images only in 2^n directions [16] and in six fixed directions [11] respectively, and these numbers of decomposing directions are both not optimal for phase information.

In this paper, we preliminarily focus on the structure of complex wavelet transforms in the implementation of relative phase. We develop a new complex wavelet transform in which subbands in the relative phase model are accurate enough in multiscale and multi-direction. Derived from the discrete shearlet transform [19,20] which has been used in some applications such as magnetic resonance imaging (MRI) reconstruction [21], we develop a new dual-channel multiscale decomposition named dual tree shearlets. It has the properties of uniform downsampled subbands, the arbitrary number of directions for decomposing and 2-D Hilbert transform relationship between its two channels like DTCWT. We compare the texture retrieval performances of relative phase of DTCWT, PDTDFB and of our dual tree shearlets.

The outline of the rest of this paper is as follows. The relative phase, the related multiresolution and multidirection complex-valued transforms are reviewed in Section 2. In Section 3, firstly we discuss in detail the requirements of relative phase and the properties of directional subbands in DTCWT and PDTDFB, and then we propose a new kind of complex shearlet transform named as dual tree shearlets, with the properties of dual-channel, multiscale, uniform downsampling along row and column in subbands and the arbitrary number of directions for decomposing. A performances comparison of the relative phase using DTCWT, PDTDFB and our dual tree shearlets in texture image retrieval is presented in Section 4. Finally, we conclude this paper in Section 5.

2. Background

2.1. Relative phase

In [7], the authors have pointed out that the local phase varies linearly with distance from features, and recently the proof for this relationship was given in [15]. For an ideal complex filter with one-side frequency support, it has been proven that the phase in the vicinity of the features such as a step or a ramp has a linear relationship with the distance in the condition of $|wt| \ll 1$, in which w is the frequency in the support of complex filter and t is the distance to feature. We refer [14] for the deep understanding of the relative phase. The relationship between the angle of an edge and the adjacent coefficients located

Table 1
The comparison of PDTDFB and the dual tree shearlets with features extracted from different number of scale in the VisTex database.

Vector	Level	Method	RP-mean (%)	RP-var (%)	RP (%)	RP-MAG (%)	MAG (%)
[8, 8, 8]	S=1	PDTDFB	56.17	43.41	**63.27**	**76.84**	65.35
		D-Shearlets	45.62	**60.13**	62.76	75.56	63.88
	S=2	PDTDFB	58.87	45.11	63.52	77.87	72.00
		D-Shearlets	62.69	**72.09**	**73.82**	**79.79**	69.85
	S=3	PDTDFB	56.54	44.34	61.13	78.81	74.60
		D-Shearlets	64.78	**73.44**	**75.48**	**80.70**	73.43
[16, 16, 16]	S=1	PDTDFB	35.08	31.98	43.58	70.33	66.09
		D-Shearlets	50.50	**62.16**	**63.85**	**75.73**	64.40
	S=2	PDTDFB	37.09	33.97	44.76	72.88	72.20
		D-Shearlets	65.31	**70.51**	**73.24**	**79.57**	70.07
	S=3	PDTDFB	36.32	32.79	42.97	74.21	74.22
		D-Shearlets	65.72	**70.60**	**73.71**	**80.20**	73.26
[4, 8, 16]	S=1	PDTDFB	35.08	31.98	43.58	70.33	66.09
		D-Shearlets	50.50	**62.16**	**63.85**	**75.73**	64.40
	S=2	PDTDFB	45.98	38.97	53.24	75.21	72.59
		D-Shearlets	62.91	**71.46**	**73.17**	**79.32**	70.07
	S=3	PDTDFB	51.03	43.95	58.53	78.35	75.10
		D-Shearlets	67.87	**73.02**	**75.53**	**81.37**	73.35