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Robust adaptive beamforming using an iterative FFT algorithm

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ABSTRACT

Adaptive beamformers will degrade in the presence of model mismatch. Because a wider beamwidth has higher ability against steering vector errors, and lower sidelobe levels can improve the robustness against fast moving interferences, in this work an iterative fast Fourier transform (FFT) based adaptive beamformer is proposed with constraints on beamwidth and peak sidelobe level. The adaptive beamforming is transformed to a weighted pattern synthesis problem. This weighted pattern is a product of the array pattern and a weighting function. Because the weighting function has shape peaks at the direction of interferences, it will have nulls in the array pattern at the directions of interferences by reducing the peak sidelobe level of this weighted pattern. A modified iterative FFT algorithm is proposed to synthesize this weighted pattern. Thanks to the efficiency of FFT, the nonconvex problem of power pattern synthesis can be solved efficiently. This method is demonstrated through several simulation examples. The results show the advantages of the proposed method in obtaining high output SINRs against moving target signals and steering vector errors.

1. Introduction

Adaptive beamforming has a wide range of applications in radar, sonar, wireless communications, medical imaging and other fields. The minimum variance distortionless response (MVDR) beamformer has superior interference rejection capability compared with the data-independent beamformers as long as the array model is known accurately [1]. In real world applications, model mismatch which can be caused by steering direction errors, imperfect array calibration, small sample size, the presence of desired signal component in the training data etc., are usually unavoidable. These imperfections will cause steering vector errors and interference-plus-noise covariance matrix cannot be estimated accurately. The performance of adaptive beamformers degrades with these imperfections. Thus, various robust adaptive beamforming techniques have been proposed in the past decades ([2–21], and many references therein). One popular and very effective approach to process the steering vector error is based on the principle of worst-case performance optimization [4]. This approach requires the steering vector error upper bound which is not known in many real world applications. Since the interference-plus-noise covariance matrix is unavailable in practice, it is usually replaced by the sample covariance matrix derived from received samples of the array output. However, the output signal-to-interference-plus-noise ratio (SINR) of MVDR beamformers with sample covariance matrix will be far from the optimal ones at the high SNRs when there is mismatch between the actual and presumed signal steering vectors or the number of snapshots is relatively small. Aiming to solve this problem, [12] proposed a robust adaptive beamforming method based on interference covariance matrix reconstruction. The output SINRs of the method in [12] are always close to optimal ones in a very large range of SNR when the array model is exactly known. However, this algorithm cannot control the beamwidth and the sidelobe levels.

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If there are desired sources moving so fast that the array weights are unable to adapt fast enough, one may prefer to design an array beampattern with constant magnitude over a wide angle region. However, the beamforming with pattern magnitude constraints is a non-convex problem which is very time consuming. For a uniform linear array, this problem can be transformed into a convex problem with the transformation from the array output power and the magnitude response to linear functions of the autocorrelation sequence of the array weight [14,16]. Because the magnitude response is not smaller than 0 at all angles in [0, 2π], enough inequality constraints on the magnitude response of different angels should be added in the optimization process (i.e. K in equation (21) of [14] should be large). If the number of sampling angles is small, the optimized autocorrelation sequence may not be an autocorrelation sequence at all. This makes this kind of technique have higher computational complexity compared with conventional robust adaptive beamforming, especially when the array is large. Most existing robust adaptive beamformers do not consider the sidelobe of the beampattern. If the beampattern has high sidelobes, it will degrade the performance greatly during the time interval of updating the weights when new interferences suddenly appear [18]. A second-order cone programming approach in [18] was proposed to control the sidelobe levels. However, this method did not consider the steering vector error which can severely degrade the performance.

An easy and promising iterative fast Fourier transform (FFT) method [22,23] was presented to synthesize large planar arrays, and it was later extended to synthesize nonuniform arrays [24] and uniform planar arrays with flat-top pattern [25]. This method is based on the Fourier transform-pair relationship between the array excitations and the array factor. Due to the efficiency of FFT, this method can solve large array synthesis problems efficiently. Here, we modify this iterative FFT method to optimize the weights of uniform linear arrays for power pattern synthesis.

In this paper, we will propose a robust adaptive beamformer based on a modified iterative FFT method for uniform linear arrays which can combine with constraints on the mainlobe beamwidth and sidelobe levels. First, we transfer the adaptive beamforming problem into a weighted array pattern synthesis problem, where the weighted pattern is a product of the array pattern and a weighting function. The value of the weighting function is equal to the Capon spectrum in the sidelobe area and is equal to a small constant in the mainlobe area. Therefore, for the weighting function, there are peaks at the directions of interferences and no peak at the direction of the target signal. Minimizing the output power of interferences and noise will be equal to minimizing the sidelobe levels of this weighted pattern. When we decrease the peak sidelobe level of the weighted pattern, the interferences are suppressed with high priority due to the great values of the weighting function at their direction of arrivals. Meanwhile we restrict the array has nearly constant magnitude response in a range of directions, which can improve the robustness of the beamformer against array steering vector errors. We adjust the array pattern by a modified iterative FFT method according to a weighted array pattern. When the weighted pattern is arrived at the desired one, the interferences have been suppressed and the array pattern satisfies the given constraints. Finally, the proposed method is validated by some simulation cases.

2. The proposed robust adaptive beamforming

Consider a uniform linear array with N isotropic sensors equally spaced at distance d that receives signals from multiple narrowband sources. The observation signal vector $\boldsymbol{x}(t)$ at the time instant t is an $N\times 1$ vector given as

$$\boldsymbol{x}(t)=\boldsymbol{s}(t)\boldsymbol{a}(u_o)+\boldsymbol{v}(t) \tag{1}$$

where $\boldsymbol{s}(t)$ is the waveform of the desired signal, $\boldsymbol{a}(u)=[1,e^{j2\pi du/\lambda},\ \ldots,\ e^{j2\pi(N-1)du/\lambda}]^T$, $u=\sin(\theta)$, u_o is associated with the target signal impinging on the array from direction θ_o, $\boldsymbol{v}(t)$ denotes the sum of the interferences and the noise. The output of beamformer is given as $\boldsymbol{y}(t)=\boldsymbol{w}^H\boldsymbol{x}(t)$, where $\boldsymbol{w}$ is the $N\times 1$ complex weight vector and $(\bullet)^H$ stands for the Hermitian transpose.

The goal of an adaptive beamformer is to receive the desired signal and reject the interferences as well as noise. With MVDR beamformer, the optimal weight vector $\boldsymbol{w}$ can be obtained by solving the following problem

$$\min_{w}\ \boldsymbol{w}^H\boldsymbol{R}_v\boldsymbol{w}\ \text{subject to}\ \boldsymbol{w}^H\boldsymbol{a}(u_o)=1 \tag{2}$$

where $\boldsymbol{R}_v=E\{\boldsymbol{v}(t)\boldsymbol{v}^H(t)\}$ is the interference-plus-noise covariance matrix. The beamformer based on (2) is sensitive to the steering vector error which can come from direction of arrival mismatch, array calibration errors etc. To make the beamformer more robust to steering vector error, we constrain the array pattern gain over a wide direction range to be greater than a given constant. Moreover, we maintain low sidelobes against unexpected interferences. The problem (2) can be rewritten as

$$\min_{w}\ \boldsymbol{w}^H\boldsymbol{R}_v\boldsymbol{w} \tag{3a}$$

$$\text{subject to}\ \eta\le|\boldsymbol{w}^H\boldsymbol{a}(u)|\le 1,\ u\in U_m \tag{3b}$$

$$|\boldsymbol{w}^H\boldsymbol{a}(u)|\le\varepsilon,\ u\in U_s \tag{3c}$$

where η and ε are given constants representing the ripple level in the mainlobe and the peak sidelobe level in the sidelobe region, U_m and U_s represent the set of u in the mainlobe region and sidelobe region, respectively.

The constraint (3b) is non-convex which is time consuming to solve. In [14], by changing the variables $\boldsymbol{w}$ into the variables $\boldsymbol{r}$ which is defined as the autocorrelation sequence of complex weights, the non-convex constraints are transformed to convex ones. However, as Section 3.4 will show, the number of angular samples should be large enough to guarantee the variable $\boldsymbol{r}$ is an autocorrelation sequence. This will increase the computational complexity especially for the large arrays. In the following, we will propose a modified iterative FFT based method to solve (3a)–(3c) efficiently.