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Robust adaptive beamforming based on interference covariance matrix sparse reconstruction

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ABSTRACT

Adaptive beamformers are sensitive to model mismatch, especially when the desired signal is present in the training data. In this paper, we reconstruct the interference-plus-noise covariance matrix in a sparse way, instead of searching for an optimal diagonal loading factor for the sample covariance matrix. Using sparsity, the interference covariance matrix can be reconstructed as a weighted sum of the outer products of the interference steering vectors, the coefficients of which can be estimated from a compressive sensing (CS) problem. In contrast to previous works, the proposed CS problem can be effectively solved by use of *a priori* information instead of using l_1-norm relaxation or other approximation algorithms. Simulation results demonstrate that the performance of the proposed adaptive beamformer is almost always equal to the optimal value.

1. Introduction

Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of adaptive spatial filtering and interference suppression. It has been widely used in radar, sonar, seismology, radio astronomy, wireless communications, acoustics, medical imaging, and other areas [1,2]. When there is no required knowledge of direction, blind source separation based beamforming tries to recover the source signals relying on the properties of the signals, such as the constant modulus especially in wireless communication [3,4] (see also Chapter 6 of [2] and the references therein). Instead, when the directions of the source signals are available, the Capon adaptive beamformer is an optimal spatial filter that maximizes the array output signal-to-interference-plus-noise ratio (SINR) [1]. However, it is also known to be sensitive to model mismatch, especially when the desired signal is present in the training data. In such a case, the Capon beamformer suffers severe performance degradation. In addition, in practical applications the required interference-plus-noise covariance matrix cannot be perfectly estimated due to the limited training samples. Therefore, adaptive beamforming approaches must be robust against covariance matrix uncertainty.

Diagonal loading is a simple and well-known robust adaptive beamforming technique [5]. However, there is no clear guideline to choose an optimal loading factor in different scenarios. Worst-case performance optimization [6,7] can also be regarded as a diagonal loading technique; however, the worst case does not always occur, and the norm upper-bound of the mismatch vector is usually *a priori* unknown. Hence, worst-case optimization is still suboptimal. In the past years, some user parameter-free adaptive beamforming algorithms were proposed (see, for example, [8], and the references therein). Unfortunately, these techniques obtain estimates of the theoretical covariance matrix of the received signal, instead of the required interference-plus-noise covariance matrix. More recently, covariance matrix reconstruction methods were proposed [9,10]. In [9], the covariance matrix was reconstructed by locating the nulls of the beampattern of the Capon

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beamformer. However, all interference powers are set to the largest eigenvalue of the sample covariance matrix, which is not optimal. In addition, the number of sources is also difficult to determine. In [10], the covariance matrix was reconstructed based on the Capon spatial spectrum, which usually underestimates the interference powers. Furthermore, the computational complexity is comparatively large because of the integral operation.

Considering the fact that the number of sources is typically less than the number of sensors in array signal processing, in this paper we reconstruct the interference-plus-noise covariance matrix in a sparse way. The reconstructed interference covariance matrix is a linear combination of the outer products of the interference steering vectors weighted by their individual powers, which can be estimated from a compressive sensing (CS) problem. This approach allows the desired signal to be removed out from the covariance matrix reconstruction; hence, there will be no signal component in the reconstructed covariance matrix, which mitigates the signal self-nulling problem. In the last decade, many signal recovery algorithms were proposed in the field of CS, such as l_1-norm convex relaxation [11,12] and greedy iterative algorithm [13] (see also [14] and the references therein). Unlike the previous works which mainly exploited the sparsity or compressibility, the proposed CS problem in this paper can be effectively solved by use of *a priori* information of the directions of the source signals, which can be estimated in advance. And hence, a closed-form solution of the CS problem can be derived. Numerical examples demonstrate that the performance of the proposed adaptive beamforming algorithm is nearly equal to the optimal value over a wide range of signal-to-noise ratios (SNRs). Meanwhile, the technique has low computational complexity.

2. The signal model

The output of a narrowband adaptive beamformer with M omni-directional sensors at time k is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k), \tag{1}$$

where $\mathbf{w} = [w_1, \ldots, w_M]^T \in \mathcal{C}^M$ is the beamformer weight vector, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. The array received vector $\mathbf{x}(k) = [x_1(k), \ldots, x_M(k)]^T \in \mathcal{C}^M$ can be represented as

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k), \tag{2}$$

where $\mathbf{x}_s(k) = \mathbf{a}s(k)$, $\mathbf{x}_i(k)$, and $\mathbf{x}_n(k)$ are statistically independent components of the desired signal, interference, and noise, respectively. In the desired signal term, $\mathbf{a} \in \mathcal{C}^M$ is the spatial steering vector of the signal waveform $s(k)$.

The optimal weight vector $\mathbf{w}$ can be obtained by maximizing the beamformer output SINR as

$$\text{SINR} \triangleq \frac{E\{|\mathbf{w}^H \mathbf{x}_s|^2\}}{E\{|\mathbf{w}^H(\mathbf{x}_i + \mathbf{x}_n)|^2\}} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{3}$$

where $\sigma_s^2 \triangleq E\{|s(k)|^2\}$ is the signal power, $\mathbf{R}_{i+n} \triangleq E\{(\mathbf{x}_i(k) + \mathbf{x}_n(k))(\mathbf{x}_i(k) + \mathbf{x}_n(k))^H\} \in \mathcal{C}^{M \times M}$ is the interference-plus-noise covariance matrix, and $E\{\cdot\}$ denotes statistical expectation. The SINR maximization problem (3) is mathematically equivalent to the minimum variance distortionless response (MVDR) problem [15]:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a} = 1, \tag{4}$$

which solution

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}}{\mathbf{a}^H \mathbf{R}_{i+n}^{-1} \mathbf{a}}, \tag{5}$$

is sometimes referred to as the Capon beamformer. From this principle of MVDR, several robust adaptive beamforming algorithms have been developed and successfully applied in a wide range of areas (see [16] and the references therein).

Since the exact interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ is not easy available even in signal-free applications, it is usually substituted by the sample covariance matrix $\hat{\mathbf{R}} = 1/K \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$ with K training snapshots, and the obtained adaptive beamformer $\mathbf{w}_{SMI} = \hat{\mathbf{R}}^{-1}\mathbf{a}/\mathbf{a}^H \hat{\mathbf{R}}^{-1}\mathbf{a}$ is called the sample matrix inversion (SMI) adaptive beamformer [17]. Whenever there is a desired signal, the SMI beamformer is in essence the minimum power distortionless response (MPDR) beamformer [1] instead of the MVDR beamformer (5). As K increases, $\hat{\mathbf{R}}$ will converge to its theoretical version $\mathbf{R} = \sigma_s^2 \mathbf{a}\mathbf{a}^H + \mathbf{R}_{i+n}$, and the corresponding SINR will approach the optimal value as $K \to \infty$ under stationary and ergodic assumptions. However, when the number of snapshots K is small, the large gap between $\hat{\mathbf{R}}$ and $\mathbf{R}$ is known to dramatically affect the performance of the SMI beamformer, especially when there is a desired signal in the training samples [5,18].

In previous works, researchers have focused on finding the optimal loading factor for $\hat{\mathbf{R}}$, which inevitably results in performance degradation, especially at high SNRs (see [8] and the references therein). The main reason is that the signal is always active in any kind of diagonal loading beamformers, and its effect becomes more and more pronounced with the increase of SNR [10]. In order to avoid the self-nulling phenomenon, in this paper, we will reconstruct the desired interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ directly, rather than searching for the potential optimal diagonal loading factor.

3. The proposed algorithm

In order to reconstruct the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$, we need to know the steering vectors of all interferences and their powers, together with the noise power. When the number of interferences, their locations, and their powers are unknown, the covariance matrix $\mathbf{R}_{i+n}$ can be estimated as [10]

$$\hat{\mathbf{R}}_{i+n} = \int_{\overline{\Theta}} \hat{p}_{Capon}(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta)\, d\theta \tag{6}$$

where $\mathbf{d}(\theta)$ is the steering vector associated with a hypothetical direction θ based on the known array structure.

$$\hat{p}_{Capon}(\theta) = \frac{1}{\mathbf{d}^H(\theta) \hat{\mathbf{R}}^{-1} \mathbf{d}(\theta)} \tag{7}$$

is the Capon spatial power spectrum estimator [15], and $\overline{\Theta}$ is the complement sector of Θ. That is to say, $\overline{\Theta} \cap \Theta = \emptyset$ and