Signal Processing 96 (2014) 401–405

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

On nonlinear amplifier modeling and identification using baseband Volterra-Parafac models

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ARTICLE INFO

Article history:
Received 15 July 2013
Received in revised form
11 September 2013
Accepted 30 September 2013
Available online 11 October 2013

Keywords:
Baseband Volterra models
Nonlinear channel estimation
Volterra kernels
Power amplifiers

ABSTRACT

The baseband Volterra–Parafac model is a useful tool to represent a nonlinear communication channel with a parametric complexity reduced with respect to the full Volterra model. In this paper we include additional symmetry properties of real power amplifier kernels in the equivalent baseband Volterra-Parafac approach in order to gain a further reduction in the number of parameters. To illustrate the new proposal, the parameters of the equivalent baseband Volterra–Parafac representation for a power amplifier are estimated using the complex least mean square algorithm. Comparison of the measured amplifier output and the model prediction for the case of an orthogonal frequency division multiplexing input signal demonstrates a notable model performance.

1. Introduction

The spectral-efficient Orthogonal Frequency Division Multiplexing (OFDM) technique, which is adopted by several modern wireless and digital video broadcasting systems (LTE, WiFi, WiMAX, DVB-T, etc.) presents a high peak-to-average power and an accordingly excessive non-linear distortion. In consequence, it is necessary to develop improved modeling techniques to represent the nonlinear channel in order to linearize the transmitter. Volterra series are commonly employed in power amplifier (PA) modeling, but its main limitation is the extremely large number of coefficients necessary to describe its response. This reason motivated the search of complexity reduction techniques by assuming elementary structures, as is the case of the Wiener model. Owing to its simplicity, a linear block and a nonlinear block connected in cascade, this representation is inadequate to describe exactly a general PA, and a different structure conceived as a finite number of parallel Wiener models has been proposed by [1]. One important drawback of these approaches is the use of general nonlinear blocks which usually makes difficult the optimization process, requiring specialized algorithms with slow convergence and much higher computational complexity.

Using the symmetry property of the Volterra models, a novel approach was proposed in [2], where the authors regarded the kernels as symmetric tensors and apply the Parafac decomposition for reducing the parametric complexity [3]. The so obtained Volterra–Parafac representation can be viewed as Wiener models in parallel, with each branch formed by a FIR linear filter in cascade with a homogeneous pth-degree memoryless nonlinearity. The corresponding baseband Volterra model, more convenient to represent a nonlinear communication channel, presents the peculiarity of including kernels characterized by a double symmetry [4]. Viewed as a tensor, the $(2p-1)$th-order kernel is symmetric with respect to the first p indices on the one hand, and its last $p-1$ indices on the other hand. Therefore, it can be decomposed using a doubly symmetric Parafac decomposition with two factor matrices to produce a baseband Volterra–Parafac (VP) model which exhibit a reduced parametric complexity.

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http://dx.doi.org/10.1016/j.sigpro.2013.09.028

In this communication, we introduce a modification to the VP model that takes into account additional symmetry properties of the equivalent low-pass kernels. Section 2 is dedicated to demonstrate that the entries of the two factor matrices are related by a complex conjugated operation, thus allowing an extra complexity reduction to nearly a half of the coefficients. The identification procedure and the validation of the model are addressed in Sections 3 and 4. To illustrate experimentally the complexity reduction, the input and output signals of a real PA are used in the validation process of the new approach, and comparison of its capability with that of the original baseband VP model is performed.

2. Modified baseband Volterra–Parafac model

The discrete-time output complex envelope y_k of a communication PA can be related to the input complex envelope x_k by a baseband Volterra model [5,6], as

$$y_k = \sum_{p=1}^{P} \sum_{q_1=0}^{Q_1} \cdots \sum_{q_{2p-1}=0}^{Q_{2p-1}} h^{(2p-1)}_{q_1,\ldots,q_{2p-1}} \prod_{i=1}^{p} x_{k-q_i} \prod_{i=p+1}^{2p-1} x^*_{k-q_i}, \tag{1}$$

where $h^{(2p-1)}_{q_1,\ldots,q_{2p-1}}$ represents the (2p–1)th-order Volterra kernel coefficients, $2P-1$ is the nonlinearity degree of the Volterra model and $Q_{2p-1}+1$ is the memory of the (2p–1) th-order homogeneous term. The equivalent baseband VP model for nonlinear communication channels was introduced by Bouilloc and Favier exploiting the complexity reduction of the Volterra kernels viewed as tensors [4]. That baseband VP model is written as

$$y_k = \sum_{p=1}^{P} \sum_{r=1}^{R_{2p-1}} (\mathbf{x}_k^{(2p-1)T} \mathbf{a}_r^{(2p-1)})^p (\mathbf{x}_k^{(2p-1)H} \mathbf{b}_r^{(2p-1)})^{p-1}, \tag{2}$$

with $\mathbf{x}_k^{(2p-1)} = [x_k, x_{k-1}, \ldots, x_{k-Q_{2p-1}}]^T$, R_{2p-1} is the symmetric rank of the Volterra kernel, and $R_1 = 1$, $\mathbf{a}_1^1 = h_{1;q_1}$. The vectors $\mathbf{a}_r^{(2p-1)}$ and $\mathbf{b}_r^{(2p-1)}$ are the rth column of the factor matrices with complex-valued entries related to the Volterra kernels by

$$h^{(2p-1)}_{q_1 q_2 \ldots q_{2p-1}} = \sum_{r=1}^{R_{2p-1}} \prod_{i=1}^{p} a^{(2p-1)}_{q_i,r} \prod_{j=p+1}^{2p-1} b^{(2p-1)}_{q_j,r}. \tag{3}$$

Eq. (2) can be simplified further if we recall the relationship between the passband Volterra kernel $\tilde{h}^{(2p-1)}_{q_1,\ldots,q_{2p-1}}$ and its baseband equivalent $h^{(2p-1)}_{q_1,\ldots,q_{2p-1}}$ [7], for which we obtain in discrete-time formulation

$$h^{(2p-1)}_{q_1,\ldots,q_{2p-1}} = \frac{1}{2^{2p-2}} \binom{2p-1}{p} \tilde{h}^{(2p-1)}_{q_1,\ldots,q_{2p-1}} \prod_{i=1}^{p} \left(e^{-j\Omega_0 q_i}\right) \prod_{j=p+1}^{2p-1} \left(e^{j\Omega_0 q_j}\right), \tag{4}$$

where $\Omega_0 = 2\pi f_0 t_s$ if f_0 is the carrier frequency and t_s is the sampling time. The coefficient of the nth-order passband Volterra kernel can be viewed as an element of a symmetric multidimensional array of order n, with real-valued elements. In that case, the nth-order Volterra kernel, with $n > 1$, can be decomposed using the symmetric Parafac model [2,3]

$$\tilde{h}^{(n)}_{q_1,\ldots,q_n} = \sum_{r=1}^{R_n} \prod_{i=1}^{n} g^{(n)}_{q_i,r}, \tag{5}$$

with $g^{(n)}_{q_i,r}$ real-valued. Substituting in (4), we obtain

$$h^{(2p-1)}_{q_1,\ldots,q_{2p-1}} = \frac{1}{2^{2p-2}} \binom{2p-1}{p} \sum_{r=1}^{R_{2p-1}} \prod_{i=1}^{p} \left(g^{(2p-1)}_{q_i,r} e^{-j\Omega_0 q_i}\right) \times \prod_{j=p+1}^{2p-1} \left(g^{(2p-1)}_{q_j,r} e^{j\Omega_0 q_j}\right). \tag{6}$$

Comparing (6) and (3) we observe that

$$b^{(2p-1)}_{q_j,r} = (a^{(2p-1)}_{q_j,r})^*, \tag{7}$$

a result that allows a reduction in the number of parameters in the input–output relationship (2). After substitution, the modified Volterra–Parafac model is given by

$$y_k = \sum_{p=1}^{P} \sum_{r=1}^{R_{2p-1}} |\mathbf{x}_k^{(2p-1)T} \mathbf{a}_r^{(2p-1)}|^{2p-2} \mathbf{x}_k^{(2p-1)^T} \mathbf{a}_r^{(2p-1)}. \tag{8}$$

As was demonstrated in [2], the passband Volterra–Parafac model is a parallel cascade Wiener model for which the FIR linear filters are given by the columns of the matrix factors of the kernels Parafac decompositions, and the static nonlinearities are simple powers of $n = 2p-1$. In the same form, the equivalent baseband VP model (8) can be easily understood as an arrangement of parallel static and homogeneous nonlinearities with inputs given by the filtered complex envelopes $\mathbf{x}_k^{(2p-1)^T} \mathbf{a}_r^{(2p-1)}$.

With respect to the model complexity, observe that the number of parameters of the modified VP (MVP) model is equal to $N_{MVP} = Q_1 + \sum_{p=2}^{P} Q_{2p-1} r_{2p-1}$ which is favorably compared to $N_{VP} = Q_1 + 2\sum_{p=2}^{P} Q_{2p-1}\, r_{2p-1}$ for the earliest model [4].

3. The algorithm for parameter estimation

With reference to model (8), and considering that $R_1 = 1$ and $\mathbf{a}_1^{(1)} = h_{1;q_1}$, we propose the use of the complex least mean square (CLMS) algorithm [8] adapted to the proposed structure with complex-valued coefficients. Then we can define the parameter vector

$$\boldsymbol{\theta} = [\boldsymbol{\theta}^{(1)T}, \ldots, \boldsymbol{\theta}^{(2P-1)T}]^T, \tag{9}$$

with

$$\boldsymbol{\theta}^{(1)} = \mathbf{a}_1^{(1)} \in \mathbb{C}^{(Q_1+1)\times 1} \tag{10}$$

and

$$\boldsymbol{\theta}^{(2p-1)} = \mathrm{vec}([\mathbf{a}_1^{(2p-1)}, \ldots, \mathbf{a}_{R_{2p-1}}^{(2p-1)}]) \in \mathbb{C}^{(Q_{2p-1}+1)R_{2p-1}\times 1}, \tag{11}$$

for $p = 2, \ldots, P$. The operator vec(.) forms a vector by stacking the column vectors of its argument. In that case, the input–output relationship (8) can be rewritten as

$$y(k) = f(k, \boldsymbol{\theta}). \tag{12}$$

If $s(k)$ is the output of the system to be identified, we can write

$$s(k) = f(k, \boldsymbol{\theta}) + e(k). \tag{13}$$

The CLMS algorithm minimizes the cost function

$$J(k) = \tfrac{1}{2}|e(k)|^2 = \tfrac{1}{2} e(k) e^*(k). \tag{14}$$