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Multidimensional prewhitening for enhanced signal reconstruction and parameter estimation in colored noise with Kronecker correlation structure

João Paulo C.L. da Costa [a], Kefei Liu [b,*], Hing Cheung So [b], Stefanie Schwarz [c], Martin Haardt [d], Florian Römer [d]

[a] Department of Electrical Engineering, University of Brasília, Brasília, Brazil
[b] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong
[c] Faculty of Electrical Engineering, Technische Universität München, Munich, Germany
[d] Communications Research Laboratory, Ilmenau University of Technology, Ilmenau, Germany

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ABSTRACT

Parameter estimation of multidimensional data in the presence of colored noise or interference with a Kronecker product covariance structure, which appears in electroencephalogram/magnetoencephalogram and multiple-input multiple-output applications, is addressed. In order to improve the accuracy of the multidimensional subspace-based estimation techniques designed for white noise, prewhitening algorithms are devised by exploiting the Kronecker structure of the noise covariance matrix. We first contribute to the development of the multidimensional prewhitening (MD-PWT) scheme which assumes that noise-only measurements are available. By applying prewhitening sequentially along various dimensions using the corresponding correlation factors estimated from the noise-only measurements, the MD-PWT significantly improves the performance of the closed-form parallel factor decomposition based parameter estimator (CFP-PE) with a small number of noise-only snapshots. When noise-only measurements are unavailable, an iterative joint estimation of noise and signal parameters and prewhitening algorithm is proposed by iteratively applying the MD-PWT and CFP-PE. Adaptive convergence thresholds are designed as the stopping conditions such that the optimal number of iterations is automatically determined. Simulation results show that the iterative scheme performs nearly the same as the MD-PWT with noise statistics, in all scenarios except for a special one of intermediate signal-to-noise ratios and high noise correlation levels.

1. Introduction

Accurate parameter estimation from R-dimensional (R-D) noisy measurements, where $R \geq 2$, is a task required for a variety of applications including radar, sonar, mobile communications, medical imaging, and seismology. The dimensions of the measured data can correspond to time, frequency, polarization, as well as the spatial dimensions such as one- or two-dimensional arrays at the transmitter and receiver. Correspondingly, the extracted parameters such as direction-of-arrival (DOA), direction-of-departure (DOD), time-of-arrival (TOA), and Doppler frequency, are of geometrical and physical nature. R-D subspace approaches to parameter estimation include 2-D MODE [1,2], 2-D unitary ESPRIT [3], unitary tensor-ESPRIT [4–6], R-D MUSIC [7], multidimensional folding (MDF) [8], improved MDF [9], and R-D rank

* Corresponding author. Tel.: +852 3442 2021; fax: +852 3442 0562.
E-mail addresses: jpdacosta@unb.br (J.P.C.L. da Costa), kefeilau@gmail.com, kefeiliu@ymail.com (K. Liu), hcso@ee.cityu.edu.hk (H.C. So), stefanie.schwarz@mytum.de (S. Schwarz), martin.haardt@tu-ilmenau.de (M. Haardt), florian.roemer@tu-ilmenau.de (F. Römer).

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reduction estimator (RARE) [10]. By taking into account the rich multidimensional structure, these methods provide superior estimation performance over their 1-D counterparts in terms of resolution, accuracy, robustness and identifiability.

In many applications, parameter estimation in the presence of colored noise or interference is required. Since signal information is extracted based on its correlation and colored noise samples are correlated, their presence seriously affects the signal parameter estimation performance. In particular, for strongly correlated noise, most of the noise energy is contained in the noise subspace spanned by a few eigenvectors associated with the largest noise eigenvalues. In case the dominant noise eigenvalues exceed the signal ones, the noise subspace is detected instead of the signal subspace. Prewhitening decorrelates the noise samples, so that the noise power is uniformly distributed over the whole noise subspace. Therefore, the disturbance between the noise and signal subspaces can be minimized and performance degradation will be avoided.

This paper covers multidimensional prewhitening techniques that exploit the Kronecker structure of the noise correlation matrix. Colored noise or interference with a Kronecker structure appears in a number of applications. For example, in electroencephalogram (EEG) and magnetoencephalogram (MEG) applications [11–14], the noise is correlated in both space and time dimensions, and a noise model which combines these two correlation matrices using the Kronecker product can well fit the noise measurements. In multiple-input multiple-output (MIMO) systems, the covariance matrix of the channel coefficients are often modeled as the Kronecker product of the receive and transmit covariance matrices [15,16]. The intentional interference signals, under certain correlation conditions, e.g., they are independent across space and correlated across time, have a covariance matrix with a Kronecker structure [17]. Moreover, the Kronecker structured noise model [18], as a form of parametric noise model [19–27], has practical and potential applications in other fields particularly in sensor array processing [28] due to its efficiency in noise characterization and computation [29]. In the nonparametric noise model, the noise is characterized by numerous parameters. This means that a large number of samples are required to estimate the noise features (e.g., noise covariance), which entails a heavy computational load and renders it difficult to distinguish the noise from the useful signals. By imposing the parametric structure on the noise, it allows to reduce the number of parameters needed to characterize the noise, leading to more robust and accurate signal and noise estimation as well as a reduced algorithm complexity.

Tensor based multidimensional parameter estimation techniques are distinguished by the tensor decompositions, which can be a higher-order singular value decomposition (HOSVD) or a PARAllel FACtor analysis (PARAFAC) decomposition. Sprawling from these newly found tensor based parameter estimation techniques, we have proposed a multidimensional prewhitening scheme called sequential generalized singular value decomposition (S-GSVD) for the case with noise-only snapshots in [30] and the iterative S-GSVD (I-S-GSVD) in [31] when noise-only snapshots are unavailable. Both works show that, the combination of R-D standard tensor-ESPRIT with the S-GSVD and I-S-GSVD results in an improved estimation accuracy over matrix based prewhitening schemes.

The contributions of this paper are summarized as follows. First, the S-GSVD assumes that the colored noise is uncorrelated in the temporal dimension. In this paper, we relax this assumption and generalize the multidimensional prewhitening technique to the case where the noise correlation can also be present in the temporal dimension. Moreover, the generalized multidimensional prewhitening (MD-PWT) simplifies the prewhitening process of S-GSVD by removing the generalized SVD (GSVD) step and hence is computationally more efficient. The computational efficiency of MD-PWT is mathematically demonstrated by comparing its required number of floating-point operations (flops) with that required in matrix based prewhitening. The MD-PWT can be integrated with both HOSVD and PARAFAC decomposition based parameter estimation schemes. In particular, the closed-form PARAFAC (CFP) [32] based parameter estimator (CFP-PE) is considered in this paper due to its applicability to arbitrary outer product based arrays (OPAs) and robustness to array spacing errors (ASEs) as well as its computational efficiency relative to the traditional alternating least squares (ALS) algorithm for the PARAFAC decomposition. Simulation results show that the MD-PWT with only a small number of noise snapshots improves remarkably the estimation accuracy of the CFP-PE, while retaining its advantage, namely, its applicability to OPA geometries and robustness to arrays with spacing errors. Second, to tackle the case when noise-only snapshots are unavailable, we propose the iterative multidimensional prewhitening (I-MD-PWT) scheme where estimation of noise and signal parameters and prewhitening are jointly performed by implementing the MD-PWT and CFP-PE in an iterative way. To reduce the computational complexity, adaptive convergence thresholds are designed as the stopping conditions for the iterative algorithm, and they change adaptively with the number of sources, estimated signal and noise powers, such that the optimal number of iterations can be automatically determined. The I-MD-PWT achieves an accurate estimation of the signal parameters and noise variance. It attains the same performance as MD-PWT, which utilizes an equal number of noise-only and signal-bearing snapshots, at low and high signal-to-noise ratios (SNRs). Nevertheless, when the noise power is comparable to the signal powers and there exists a fairly strong correlation between noise samples, the I-MD-PWT is inferior to its non-iterative counterpart.

The remainder of the paper is organized as follows. In Section 2, after introducing the notation used in this paper, the R-D parameter estimation problem is formulated using the PARAFAC data contaminated by Kronecker structured colored noise model. In Section 3, classical matrix and vector based techniques for prewhitening R-D noisy measurements are reviewed. In Section 4, we propose the MD-PWT technique and its iterative version, namely I-MD-PWT. In Section 6, extensive simulation results are presented to evaluate the performance of the proposed multidimensional prewhitening techniques. Finally, conclusions are drawn in Section 7.

2. Data model

In order to facilitate the distinction between scalars, matrices, and tensors, the following notation is used: Scalars