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Gaussian mixture CPHD filter with gating technique

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ABSTRACT

Cardinalized probability hypothesis density (CPHD) filter provides more accurate estimates of target number than the probability hypothesis density (PHD) filter, and hence, also of the states of targets. This additional capability comes at the price of greater computational complexity: $O(NM^3)$, where N is the number of targets and M is the cardinality of measurement set at each time index. It is shown that the computational cost of CPHD filter can be reduced by means of reducing the cardinality of measurement set. In practice, the cardinality of measurement set can be reduced by gating techniques as done in traditional tracking algorithms. In this paper, we develop a method of reducing the computational cost of Gaussian mixture CPHD filter by incorporating the elliptical gating technique. Computer simulation results show that the computational cost is reduced and that the tracking performance loss incurred is not significant.

1. Introduction

The random finite set (RFS) approach [1,5,6,10,12,15,17,18,26] to multitarget tracking has attracted much attention in recent years. In the RFS formulation for multitarget tracking, multitarget state and multitarget measurement are naturally modeled by RFS. So, the mathematical tools provided by finite set statistics (FISST) [1] can be used to extend the Bayesian inference to multitarget tracking problems. Compared with the traditional association-based mutlitarget tracking approaches, the difficulties caused by data association is avoided. However, the multitarget Bayes recursion proposed by Mahler is intractable in most practical applications [5]. The probability hypothesis density (PHD), which is the first moment associated with the multitarget posterior, was proposed by Mahler in [5]. The PHD filter is a more tractable alternative to optimal multitarget filtering. Since the domain of the intensity function is the single-target state space, its propagation requires much less computation than the multitarget posterior. The implementations of PHD filter include sequential Monte Carlo PHD (SMC-PHD) [7] and Gaussian mixture PHD (GM-PHD) [6,13]. Especially, the GM-PHD is a promising implementation with easy peak extraction. The GM-PHD filter for nonlinear dynamical models was introduced in [22]. A generalization called cardinalized probability hypothesis density (CPHD) [10,11,14] filter is introduced by Mahler. The CPHD filter differs from the PHD filter in that, in addition to the PHD, it propagates the entire probability distribution on target number. It also admits more general false alarm models: i.i.d. (independent, identical distribution) cluster processes [17,23] rather than Poisson processes. It provides more accurate estimates of target number than the PHD filter, and hence also of the states of the targets. This additional capability comes at the price of greater computational complexity: $O(NM^3)$, where N (abbreviation of N_k) is the number of targets and M (abbreviation of M_k) is the number of measurements at each time index. Recent research by Vo. et al. has indicated that a Gaussian mixture implementation of the CPHD filter outperforms JPDA in a cluttered environment

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[16,17]. Its Gaussian mixture form [17] has been successfully used to track multiple ground moving targets using road-map information in [18]. In addition, that the Gaussian mixture CPHD (GM-CPHD) filter is equivalent to the multihypothesis tracker including a sequential likelihood test for track extraction in the single target case has been proven in [18].

If we look carefully at the computational complexity of CPHD filter, we can see that the computational cost of CPHD can be reduced by means of reducing the cardinality of measurement set. In practice, the cardinality of measurement set can be reduced by gating techniques as used in traditional tracking algorithms [2–4,8,9]. In this paper, we develop a method of reducing the computational cost of the GM-CPHD filter by incorporating the elliptical gating technique. However, elliptical gating may eliminate all measurements not associated with targets that have already been detected, thus making it difficult to detect newly appearing targets. To address this, we use the CPHD filter with birth-target model to account for the possibility of newly appearing targets. Computer simulation results show that the computational cost is reduced and that the tracking performance loss incurred is not significant.

This paper is organized as follows. Section 2 provides an overview of the GM-CPHD recursion. The GM-CPHD filter with gating technique is addressed in Section 3. Demonstrations and numerical studies are considered in Section 4.

2. Gaussian mixture CPHD filter

The multitarget state and multitarget measurement based on the FISST are RFS:

$$X_k = \{x_{k,1}, x_{k,2}, \ldots, x_{k,N_k}\} \subset F(X)$$

$$Z_k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,M_k}\} \subset F(Z)$$

$$Z_{1:k} = \bigcup_{i=1}^{k} Z_i$$

where $F(X)$ and $F(Z)$ are the respective collections of all finite subset of X and Z. N_k and M_k are, respectively, the number of targets and measurement at time k.

Uncertainty in a multitarget system based on FISST is characterized by modeling the multitarget state X_k and multitarget measurement Z_k as RFS. In a similar vein to the single target dynamical model, the randomness in the multitarget evolution and observation are captured in the multitarget Markov transition density $f_{k|k-1}(\cdot|\cdot)$ and multitarget sensor likelihood function $h_k(\cdot|\cdot)$, respectively. Denote the multitarget posterior density function as $p(X_k|Z_{1:k})$, Then, the optimal multitarget Bayes filter [1,5,7] propagates the multitarget posterior in time via the recursion:

$$p(X_k|Z_{1:k-1}) = \int f_{k|k-1}(X_k|X) p(X|Z_{1:k-1}) \mu(dX) \tag{1}$$

$$p(X_k|Z_{1:k}) = \frac{h_k(Z_k|X_k) p(X_k|Z_{1:k-1})}{\int h_k(Z_k|X) p(X|Z_{1:k-1}) \mu(dX)} \tag{2}$$

where $\mu(T) = \sum_{i=0}^{\infty} \lambda^i (\chi^{-1}(T) \cap \mathrm{X}^i)/i!$ is an appropriate reference measure on $F(X)$ [1]. λ^i is the ith product Lebesque measure, and $\chi : \uplus_{i=0}^{\infty} X^i \to F(X)$ is a mapping of vector to sets defined by $\chi([x_1, \ldots, x_i]^T) = \{x_1, \ldots, x_i\}$. This reference measure is often used in point process theory.

The Bayes recursion involves multiple set integrals on the space of finite sets, which are computationally intractable [5]. The PHD filter is more tractable than the optimal multitarget filtering. It is a recursion propagating the first order statistical moment of the posterior multiple targets state. For an RFS X on X with probability distribution P, its first order moment is a nonnegative function υ on X, called the intensity or PHD. For each region $S \subset X$, the PHD has the property that [25]

$$\int |X \cap S| P(dX) = \int_S \upsilon(x)\, dx \tag{3}$$

Since the integral domain of the intensity function is the single-target state space, its propagation requires much less computational cost than the multitarget posterior. However, the PHD filter is sensitive to missed detections [24].

The CPHD recursion was proposed by Mahler in [10–12] to address the limitations [24] of the PHD recursion. The CPHD filter differs from the PHD filter in that, in addition to the PHD, it propagates the entire probability distribution function on target number. The CPHD recursion was reformulated in [17]. Based on two lemmas which were reformulated from Kalman filter in [6], a closed-form solution to the PHD recursion has been obtained in [6]. In the similar way, Gaussian mixture CPHD [16,17] filter can be obtained from the reformulated CPHD recursion too. The closed-form solution to the CPHD recursion requires assumptions of additional class of linear Gaussian multitarget models besides the assumptions of the original CPHD. The class of linear Gaussian multitarget models consists of standard linear Gaussian assumptions of the transition and observation models of individual targets, as well as certain assumptions on the birth, death and detection of targets.

Assume each target follows a linear Gaussian dynamical model, i.e.

$$f_{k|k-1}(x|\varsigma) = N(x; F_{k-1}\varsigma, Q_{k-1}) \tag{4}$$

$$g_k(z|x) = N(z; H_k x, R_k) \tag{5}$$

where $N(\cdot; m, P)$ denotes a Gaussian density with mean m and covariance P, F_{k-1} is the state transition matrix, Q_{k-1} is the process noise covariance, H_k is the observation matrix, and R_k is the measurement noise covariance, and k is the time index.

The survival and detection probabilities are assumed state independent, i.e.

$$p_{s,k}(x) = p_{s,k}, \quad p_{D,k}(x) = p_{D,k}$$

The intensity of the birth RFS at time k is a Gaussian mixture of the form

$$\gamma_k(x) = \sum_{i=1}^{J_{\gamma,k}} \omega_{\gamma,k}^{(i)} N(x; m_{\gamma,k}^{(i)}, P_{\gamma,k}^{(i)}) \tag{6}$$

where $\omega_{\gamma,k}^{(i)}$, $m_{\gamma,k}^{(i)}$ and $P_{\gamma,k}^{(i)}$ are the weights, means, and covariance matrixes of the Gaussian mixture intensity, respectively. Let $\upsilon_{k|k-1}$ and υ_k denote the intensities associated with the multitarget predicted density