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When math operations have visuospatial meanings versus purely symbolic definitions: Which solving stages and brain regions are affected?☆

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ABSTRACT

How does processing differ during purely symbolic problem solving versus when mathematical operations can be mentally associated with meaningful (here, visuospatial) referents? Learners were trained on novel math operations (↓, ↑), that were defined strictly symbolically or in terms of a visuospatial interpretation (operands mapped to dimensions of shaded areas, answer = total area). During testing (scanner session), no visuospatial representations were displayed. However, we expected visuospatially-trained learners to form mental visuospatial representations for problems, and exhibit distinct activations. Since some solution intervals were long (~10 s) and visuospatial representations might only be instantiated in some stages during solving, group differences were difficult to detect when treating the solving interval as a whole. However, an HSMM-MVPA process (Anderson and Fincham, 2014a) to parse fMRI data identified four distinct problem-solving stages in each group, dubbed: 1) encode; 2) plan; 3) compute; and 4) respond. We assessed stage-specific differences across groups. During encoding, several regions implicated in general semantic processing and/or mental imagery were more active in visuospatially-trained learners, including: bilateral supramarginal, precuneus, cuneus, parahippocampus, and left middle temporal regions. Four of these regions again emerged in the computation stage: precuneus, right supramarginal/angular, left supramarginal/inferior parietal, and left parahippocampal gyrus. Thus, mental visuospatial representations may not just inform initial problem interpretation (followed by symbolic computation), but may scaffold on-going computation. In the second stage, higher activations were found among symbolically-trained solvers in frontal regions (R. medial and inferior and L. superior) and the right angular and middle temporal gyrus. Activations in contrasting regions may shed light on solvers' degree of use of symbolic versus mental visuospatial strategies, even in absence of behavioral differences.

Introduction

Different strategies for solving a math problem can involve different types of mental representations and different neural substrates, and may have different implications for transfer and future achievement (e.g., Geary, 2011; Price et al., 2013; Pyke et al., 2015). Strategies and instructional materials involving visuospatial representations are of particular interest. Some famous mathematicians report relying heavily on mental imagery to guide their mathematical thinking (Tall, 2006; see Hadamard (1945) for a discussion of Einstein), and students' spontaneous construction and use of effective visuospatial representations can predict their math problem-solving performance (Blatto-Vallee et al., 2007; Hembree, 1992; van Garderen, 2006). Such strategies presumably contribute to correlations between spatial ability and math performance (e.g., Clements and Battista, 1992; Gathercole and Pickering, 2000; Kyttälä and Lehto, 2008; Reuhkala, 2001; for a review see Mix and Cheng (2011). That said, the use of visuospatial representations is not always helpful (Berends and van Lieshout, 2009; Booth and Koedinger, 2012; Hegarty and Kozhevnikov, 1999; Larkin and Simon, 1987; Presmeg, 1997, 2006). Such research contrasting behavioral performance across symbolic versus visuospatial strategies of various types and in various contexts is on-going (e.g., for reviews see Arcavi (2003), Hembree (1992) and Presmeg (2006)). However, far less research has investigated the neural substrates supporting visuospatial mental strategies during math problem solving. More generally, we regard visuospatial referents as a way to operationalize a more fundamental contrast of interest: between solution processes when the operations have semantic meaning versus solution processes characterized by rote calculation.

As a very simple example, one might interpret a problem like 4*5 in

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abstract symbolic terms, or in visuospatial terms, such that the answer represents the area of a rectangle of width 4 and height 5. Regardless of whether the problem is interpreted symbolically or visuospatially, one might compute the numerical answer using the same arithmetic steps (e.g., 4*5 = 4+4+4+4+4), however in the latter case the answer and operands are associated with visuospatial meanings.

The present research was motivated by two main questions. First, do neural substrates differ when learners solve math problems with purely symbolic procedures versus when learners can mentally associate the problems with meaningful – here, visuospatial - referents? Second, is the role of visuospatial and/or semantic processing restricted to specific stages during problem solving (e.g., initial interpretation vs. computation)? To explore these questions, we introduced learners to novel math operations (↓, ↑), that were defined either in terms of purely symbolic computation algorithms (symbolically-trained group) or included visuospatial referents (visuospatially-trained group). Both groups then mentally solved problems in a scanner. We then applied a relatively novel analysis processes to segment the fMRI solving-interval data into distinct mental stages (Anderson and Fincham, 2014a) – to allow us to assess whether there were stage-specific (vs. overall) neural differences across groups.

Role of visuospatial math representations: Depicting relative magnitudes

A general feature of effective visuospatial math representations (e.g., number lines, strips and graphs) is that they spatially represent the relative magnitudes of relevant quantities (e.g., as locations, lengths or areas) (e.g., Beckmann, 2004; Murata, 2008; Lewis, 1989). Prior research suggests that math learning and transfer benefit from learners having knowledge about the magnitudes of problem elements (Siegler and Ramani, 2009; Whyte and Bull, 2008), and the magnitude relations among these elements, which characterize the operation (Booth and Siegler, 2008; Slavit, 1998). For example, Booth and Siegler (2008) found that children were better able to memorize or estimate answers for specific addition facts when, during training, they had been exposed not only to the symbolic fact (e.g., 5 + 4 = 9; 18 + 16 = 34), but also to shaded bars representing the magnitudes of each operand and the sum. For arithmetic word problems, spatial representations of relative quantities with strips/bars are commonly taught and used in countries like Singapore (Beckmann, 2004; Lee et al., 2007; Lee et al., 2010) and Japan (Murata, 2008), where students exhibit high math achievement as indexed by the Programme for International Student Assessment (Organisation for Economic Co-operation and Development: OECD, 2014). Visuospatial referents have also been used in the instruction of more advanced math topics like quadratics (Hoong et al., 2010) and integral calculus, where an answer can be represented as the area of a two-dimensional region. In our experiment we investigate the impact of training learners with two–dimensional spatial referents (vs. purely symbolic procedures) on their activation patterns when they later mentally solve problems.

Neural substrates for visuospatial mental representations in math problem solving

We hypothesized that visuospatially-trained solvers might exhibit more activation in some regions to support processing the mental imagery and relational information inherent in (mental) visuospatial referents of problems. Some prior research on math cognition may shed light on which regions might be implicated in supporting such visuospatial mental referents.

One visuospatial math representation hypothesized to support numerical cognition is the *mental number line* for representing the magnitudes of symbolic numbers. The brain region most commonly associated with the mental number line is the horizontal intra-parietal sulcus (HIPS; for meta-analyses see Cohen Kadosh et al. (2008) and Dehaene et al. (2003), whose activation in is modulated by the numerical distance between numbers in a comparison task (e.g., small: 2 vs. 3; large: 2 vs. 9; Pinel, Dehaene, Riviere & LeBihan, 2001). Other studies have implicated the angular gyrus (AG) in mental number line processes (e.g., Cattaneo et al., 2009; Göbel et al., 2001). The posterior superior parietal lobule (PSPL) extending into the precuneus is also implicated in such numerical tasks (Pinel et al., 2001) and in domain-general visuospatial procesing, so Dehaene et al. (2003) suggest it may modulate attention along the mental number line. If the semantic roles of the HIPS, AG, PSPL and precuneus generalize beyond the canonical mental number line to other visuospatial math representations that convey magnitude relations, we might expect increased activation in such regions when solvers can mentally associate problems with more general visuospatial representations versus purely symbolic procedures.

Neuroscience studies exploring the use of more general and varied visuospatial math representations sometimes contrast conditions in which the problem stimuli themselves are symbolic versus visuospatial – for example: processing a sequences of quantities represented as digits versus sets of dots (or mixed, Piazza et al., 2007); adding digits versus dots (Venkatraman et al., 2005); comparing graphs versus equations (Thomas et al., 2010); and solving for relations depicted as bar lengths versus symbolic expressions (Lee et al., 2010).

Note, however, that we are ultimately interested in a slightly different type of contrast: when a solvers' *mental interpretation* may be either purely symbolic (e.g., 4+4+4+4+4) or visuospatial (area of rectangle) for the *same problem stimulus* (4*5). However, since mental imagery is known to share many neural substrates with perception (Ganis et al., 2004), neural differences in processing visuospatial versus symbolic math ***stimuli*** may foreshadow neural differences in processing visuospatial versus symbolic ***mental interpretations*** (of a common problem stimulus).

Interestingly, some studies emphasize the similarity of activation patterns in some regions across symbolic and visuospatial math stimuli. For example, Piazza et al. (2007) found that when learners saw a sequence of similar quantities (e.g., 18, 17, 19, 17, 19, 18,...) followed by a new quantity (20 or 50), the response to the new quantity in the HIPS and frontal regions depended on the difference between the new quantity and the familiar quantities (e.g., 20 is near; 50 is far). Importantly this effect was notation-independent – that is, it occurred regardless of whether or not the new quantity was displayed in the same format (digits or dots) as the quantities in the original sequence. They did however report that the fusiform gyrii and left lingual gyrus were sensitive to a format change.

Other stimulus-contrast studies report differences in other regions. For example, Thomas et al. (2010) reported greater activity when participants processed graphs (vs. corresponding linear and quadratic equations) not only in a bi-lateral occipital region but also in the right posterior superior parietal lobe (PSPL), precuneus, right postcentral gyrus, and right middle temporal gyrus. In such experimental designs where the stimuli differ across conditions, despite clever controls, it is not always clear which activation differences are just due to stimulus format differences versus distinct mental semantic and visuospatial solution processes.

Other studies have controlled stimulus format (e.g., symbolic expressions or word problems) but still found distinct activation patterns when learners used visuospatial *mental* strategies (e.g., Lee et al., 2007; Zago et al., 2001; Zago et al., 2008; Zarnhofer et al., 2013). For word problems, Zarnhofer et al. (2013) found that a measure reflecting the self-reported degree of use of mental visualization strategies was correlated in both hemispheres with activation in occipital regions, the lingual gyrus, calcarine gyrus, cuneus, and thalamus; and with right hemisphere (only) activation in the fusiform and superior, middle and inferior temporal gyrii.

Additional evidence comes from learners who can use mental abacus imagery to solve problems. When children trained on abacus use did exact mental addition in a scanner, several regions were more active than among